Numerical Analysis

MAT 542 – FALL 2010

Homework # 6 Due October 8

1. Put these differential equations into a form suitable for numerical solution by the Runge-Kutta method.

(a) $x + 2xx' - x' = 0$

(b) $\log x' = t^2 - x^2$

(c) $(x')^2(1 - t^2) = x$

(d) $x' = t^2 + tx' - 2xx'$

(e) $x' = e^t + x' \cos x + t^2$

2. Solve the differential equation

$$\begin{cases} \frac{dx}{dt} = -tx^2 \\ \\ x(0) = 2 \end{cases}$$

at $t = -0.2$, correct to two decimal places, using one step of the Taylor series method of order 2 and one step of the Runge-Kutta method of order 2.

3. Consider the ordinary differential equation

$$\begin{cases} x' = (tx)^3 - \left(\frac{x}{t}\right)^2 \\ \\ x(1) = 1 \end{cases}$$

Take one step of the Taylor series method of order 2 with $h = 0.1$ and then use the Runge-Kutta method of order 2 to recompute $x(1.1)$. Compare answers.

4. **(G)** Describe how the fourth-order Runge-Kutta method can be used to produce a table of values for the function

$$f(x) = \int_0^x e^{-t^2} dt$$

at 100 equally spaced points in the unit interval. *Hint:* Find an appropriate initial-value problem whose solution is f.