Numerical Analysis

MAT 542 – FALL 2010

Homework # 5 Due September 29

1. Check whether the given function $x(t)$ is a solution to the initial value problem (IVT):

(a)

$$\begin{cases} x' = t^2 + t^{1/3} \\ x(0) = 7 \end{cases} \qquad x(t) = \frac{1}{3}t^3 + \frac{3}{4}t^{4/3} + 7$$

(b)

$$\begin{cases} x' = 2x \\ x(0) = 15 \end{cases} \qquad x(t) = 15e^t$$

(c)

$$\begin{cases} x'' = -x \\ \\ x(\pi) = 0 \\ x'(\pi) = 3 \end{cases} \qquad x(t) = -3\sin t$$

2. Determine x'' when $x' = xt + x^3 + e^x t$.

3. Consider the IVP

$$\begin{cases} x' = x \\ x(0) = c \end{cases}$$

whose solution is $x(t) = ce^t$. If a roundoff error of ϵ occurs in reading the value of c into the computer, what effect is there on the solution at the point $t = 10$? At $t = 20$?

Repeat the process for the IVP

$$\begin{cases} x' = -x \\ x(0) = c \end{cases}$$

whose solution is $x(t) = ce^{-t}$.

3. **(G)** Calculate an approximate value for $x(0.1)$ using one step of the Taylor series method of *order 3* on the IVP

$$\begin{cases} x'' = x^2 e^t + x' \\ \\ x(0) = 1 \\ x'(0) = 2 \end{cases} .$$