Numerical Analysis

MAT 542 – FALL 2010

Homework # 3 Due September 10

1. If $p(x)$ is the polynomial

$$p(x) = a_0 + a_1 x + a_2 x^2 + ... + a_r x^r,$$

with non-negative coefficients ($a_i \geq 0,\ i = 0, 1, ..., r$) and

$$p(A) = a_0 I + a_1 A + a_2 A^2 + ... + a_r A^r,$$

show that

$$||p(A)|| \leq p(||A||).$$

2. The Jacobi and Gauss-Seidel methods are used to solve

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} x = \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix}$$

- In each case compute the iteration matrix and its spectral radius.
- Check whether the method converges if the starting solution vector is $x_0 = [10\ 100\ 1000]^T$.
- Is there an easier way to check the convergence of the two methods?

3. Consider the positive definite matrix

$$A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}.$$

Let

$$\mathbf{v}^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{v}^{(2)} = \begin{bmatrix} -\frac{3}{4} \\ 1 \\ 0 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{v}^{(3)} = \begin{bmatrix} -\frac{3}{7} \\ \frac{4}{7} \\ 1 \end{bmatrix}.$$

- Is the set $\{\mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \mathbf{v}^{(3)}\}$ an A-orthogonal (A-conjugate) set?
- For the system $Ax = b$, where $b = [24,\ 30,\ -24]^T$, find $x^{(2)}$ using the conjugate gradient method given the initial guess $x^{(0)} = [0,\ 0,\ 0]^T$.