Numerical Analysis

MAT 542 – FALL 2010

Homework # 2 Due September 3

1. Find the condition numbers of

$$A = \begin{bmatrix} 1 & 2 \\ 1.001 & 2.001 \end{bmatrix}$$

for the l_∞, and l_1 norms. This matrix is ill-conditioned because the second row is almost a multiple of the first row.

2. Show that if A and B are $n \times n$ matrices, their condition numbers satisfy the following for any choice of norm and scalar $\lambda \neq 0$.

(a) $\kappa(A) \geq 1$

(b) $\kappa(AB) \leq \kappa(A)\kappa(B)$

(c) $\kappa(\lambda A) = \kappa(A)$

3. Show that if A is non singular then

$$||A^{-1}|| \geq \frac{1}{||A||}.$$

4. Carry out three iterations of

(i) the Jacobi method

(ii) the Gauss-Seidel method

to solve

$$\begin{bmatrix} 5 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix} x = \begin{bmatrix} 7 \\ 4 \\ 5 \end{bmatrix}.$$

(Solution $x \approx [2.0870\ 3.4348\ 4.2174]^T$.)

In each case start with $x_0 = [0\ 0\ 0]^T$.