October 18, 2010

Note Title 10/18/2010

Last time:

$$\begin{cases} x' = y \\ y' = x \end{cases}$$

$x(0) = -1, \quad y(0) = 0$

using 4th order Runge-Kutta

$$k_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} -0.05 \\ -0.95 \end{bmatrix}$$

$$k_3 = F\left(t + \tfrac{1}{2}h,\ X + \tfrac{1}{2}h k_2\right)$$

$$= F\left(0 + \tfrac{1}{2}h,\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} + \tfrac{1}{2}(0.1)\begin{bmatrix} -0.05 \\ -0.95 \end{bmatrix}\right)$$

$$= \begin{bmatrix} y(0) + \tfrac{1}{2}(0.1)(-0.95) \\ x(0) + \tfrac{1}{2}(0.1)(-0.05) \end{bmatrix}$$

$$= \begin{bmatrix} -0.0475 \\ -1.0025 \end{bmatrix}$$

$$k_4 = F\left(t + h,\ X + hK_3\right)$$

$$= F\left(0 + h,\ \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} + 0.1 \begin{bmatrix} -0.0475 \\ -1.0025 \end{bmatrix}\right)$$

$$= \begin{bmatrix} y(0) + (0.1)(-1.0025) \\ x(0) + (0.1)(-0.0475) \end{bmatrix}$$

$$= \begin{bmatrix} -0.100025 \\ -1.004475 \end{bmatrix}$$

Hence $\begin{bmatrix} x(0.1) \\ y(0.1) \end{bmatrix} = \begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$+ \frac{0.1}{6}\left\{\begin{bmatrix} 0 \\ -1 \end{bmatrix} + 2\begin{bmatrix} -0.05 \\ -0.95 \end{bmatrix} + 2\begin{bmatrix} -0.0475 \\ -1.0025 \end{bmatrix} + \begin{bmatrix} -0.10025 \\ -1.00475 \end{bmatrix}\right\}$$

$x(0.1) = -1.0049208333$

$y(0.1) = -0.0984958333$

Exact values:

$x(0.1) = -\cosh(0.1) = -1.0050041668$

$y(t) = -\sinh(0.1) = -0.1001667 5$

$$\left| x(0.1) - \hat{x}(0.1) \right| = 8.333 \times 10^{-5}$$

$$\cosh t = \frac{e^t + e^{-t}}{2}$$

Definition

A system of differential equations without the t variable explicitly present is said to be autonomous.

Example

$$\begin{cases} x' = y \\ y' = x \\ x(0) = -1, \; y(0) = 0 \end{cases}$$

Autonomous

Example

$$\begin{cases} x' = x(t) - y(t) + 2t - t^2 - t^3 \\ y' = x + y - 4t^2 + t^3 \\ x(0) = 1, \quad y(0) = 0 \end{cases}$$

NOT Autonomous!

Lab 3

$$x'(t) = x + x^2$$

$$x''(t) = x' + 2x x'$$

$$x'''(t) = x'' + 2x'x' + 2xx''$$

$$= 2(x')^2 + (1+2x)x''$$

$$x^{(4)} = 4x'x'' + 2x'x'' + (1+2x)x'''$$

$$= 6x'x'' + (1+2x)x'''$$

$$x^{(5)} = 6x''x'' + 6x'x''' + 2x'x''' + (1+2x)x^{(4)}$$

$$= 6(x'')^2 + 8x'x''' + (1+2x)x^{(4)}$$