Mathematical Modeling – MAT 506/606
Fall 2013
Homework 3

Due date: September 11, 2013

1. *(8 points)* Answer the following questions related to geometric similarity and proportionality:

a. If water bottles are geometrically similar, how much more water will a bottle that is 30 cm tall hold than one that is 10 cm tall?

b. If air resistance is proportional to the surface area of a falling object at a given velocity, how much more air resistance will a sphere of diamater 5 cm encounter than one with a diameter of 1 cm?

c. If gymnasts have a constant density, then weight is proportional to volume. If we further assume that they are geometrically similar, how much less would a gymnast who is 5 ft. tall weigh than one who is 5.5 ft. tall?

d. If hearts are geometrically similar and the volume of blood pumped in one beat is proportional to the volume of the heart, how much more blood will a heart 4 cm wide pump in one beat than a heart that is 1 cm wide?

2. *(4 points)* For the data below, fit a model of the given form by transforming the data appropriately and fitting a straight line to the transformed data. Graph the residuals and analyze how well the model fits the data.

Model: $y = ax^2 + b$

x	1	2	3	4	5	6
y	16.3	23.1	37.4	46.9	58.7	91.0

3. *(4 points)* [**Graduate**] Answer the following questions related to geometric similarity and proportionality:

a. If objects are geometrically similar and have a constant density, we saw that $A \propto W^{2/3}$ where $A =$ Surface Area and $W =$ Weight. If the weight of one such object is 5 times the weight of another, how much larger is the surface area?

b. Suppose that an ice cube melts so that at any point in time, the remaining cube is geometrically similar to the initial cube (i.e. before it started melting). At one point in time, the length is half the initial length. What fraction of the initial volume has melted?

4. *(4 points)* [**Graduate**] For the data below, fit a model of the given form by transforming the data appropriately and fitting a straight line to the transformed data. Graph the residuals and analyze how well the model fits the data.

Model: $y = a\sin(x) + b$

x	1	2	3	4	5	6
y	1.34	1.61	-0.98	-3.80	-4.55	-2.30