MAT 442(01) – Spring 2014
Introduction to Numerical Analysis

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Problem Set 1

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1. Why do the following functions not posses Taylor series expansion at $x = 0$?
   - $f(x) = \sqrt{x}$
   - $f(x) = |x|$
   - $f(x) = \arcsin(x - 1)$
   - $f(x) = \cot x$
   - $f(x) = \log x$
   - $f(x) = x^{\pi}$

2. Use Horner's algorithm to deflate the polynomial $p(x) = x^4 - 4x^3 + 7x^2 - 5x - 2$ by removing the linear factor $(x - 3)$. Hence evaluate $p(3)$.

3. What is the relative error involved in rounding 0.3720214371 to five decimal digits of accuracy?

4. Determine the first two nonzero terms of the series expansion about zero for the following
   - $e^{\sin x}$
   - $\sin(\cos x)$

5. Convert the decimal numbers to binary.
   - $(256)_{10}$
   - $(0.328)_{10}$

6. Convert the binary number $(0.110101)_2$ to decimal.

7. Enumerate the set of numbers in the floating-point number system that have binary representation of the form

$$\pm(0.b_1b_2) \times 2^k, \text{ where } k \in \{-1, 1\}.$$

8. In the subtraction $0.06666666667 - 0.06661729492$, how many bits of significance will be lost?

9. How can values of the function $f(x) = \sqrt{x+4} - 2$ be computed accurately when x is small?

10. For what values of x may loss of significance occur in the computation of $f(x) = \log(x+1) - \log x$. How can that loss of significance be minimized.

11. Let $f(x) = \dfrac{1-x}{1+x} - \dfrac{1}{3x+1}$. For very small values of x, loss of significance can occur. How can you minimize loss of significance?

12. What difficulty could the following assignment cause?

$$y \leftarrow 1 - \sin x.$$

Circumvent it without resorting to a Taylor series if possible.

13. Every polynomial of degree n has n zeros (counting multiplicity) in the complex plane.

- Does every real polynomial have n real zeros?

- Does every *polynomial of infinite degree* $f(x) = \sum_{n=0}^{\infty} a_n x^n$ have infinitely many zeros?

14. State the order of convergence of the methods listed below when used to determine the zeros of a given nonlinear function $f(x)$.

- the Bisection method

- the Newton-Raphson method

- the Secant method

15. Solve the equation $x^2 - 10^5 x + 1 = 0$ with a machine that carries only eight decimal digits.

16. How many steps of the bisection method are needed to determine the root with an error of at most $\frac{1}{2} \times 10^{-12}$, if the starting interval is $[0.2, 1.8]$?

17. Compute the zero of $f(x) = x^3 - 3x + 1$ on $[0, 1]$ using the Bisection method. Carry out just three steps.

18. If Newton's method is used on $f(x) = x^3 - x + 1$ starting with $x_0 = -2$, what will x_3 be?

19. If we use the secant method on $f(x) = x^3 - 2x + 2$ starting with $x_0 = 0$ and $x_1 = 1$, what is x_3?