MAT 442(01) – Spring 2013
Introduction to Numerical Analysis

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Problem Set 4
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Name : _____

1. What is the numerical value of the Composite Trapezoid rule applied to the reciprocal function $f(x) = x^{-1}$ using the points $1, \frac{4}{3}$, and 2.

2. Compute the approximate value of $\int_0^{\pi/4} x \sin x \; dx$ by using the Composite Trapezoid rule with three points. Then compare with the actual value of the integral.

3. If the Composite Trapezoid rule is used to compute $\int_{-1}^{2} \sin x \; dx$ with $h = 0.01$, give a realistic bound on the error.

4. How large must n be if the Composite Trapezoid rule is being used to estimate $\int_0^\pi \sin x \, dx$ with error $\leq 10^{-12}$? Will the estimate be too big or too small?

5. Consider $\int_1^2 \frac{dx}{x^3}$. What is the result of using the composite trapezoid rule with the partition points 1, $\frac{3}{2}$, and 2?

6. We want to approximate $\int_1^2 f(x) \, dx$ given the table of values. Compute an estimate by the Composite Trapezoid rule. Can upper and lower sums be computed from the given data?

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
f(x)	10	8	7	6	5

7. Compute $\int_0^{0.35} \frac{2}{x^2-4} dx$ by the basic Simpson's Rule. Compare with the true solution.

8. Find an approximate value of $\int_1^2 x^{-1}\, dx$ using the basic Simpson's Rule with uniform spacing. Give a bound on the error.

9. Find an approximate value of $\int_{-2}^2 x^3 e^x\, dx$ using the Composite Simpson's Rule with uniform spacing with $n = 4$.

10. Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)\ dx \approx c_0 f(0) + c_1 f(x_1)$$

gives exact results for all polynomials of degree at most 2.

11. Approximate

$$\int_0^2 e^{-x^2}\ dx$$

using the three point Gaussian Quadrature formula

$$\int_{-1}^1 f(x)\ dx \approx \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right).$$