MAT 442(01) – SPRING 2010
Introduction to Numerical Analysis

Murray State University
Murray, KY 42071

Problem Set 4
MARCH 2010

Name : _____

1. What is the numerical value of the composite trapezoid rule applied to the reciprocal function $f(x) = x^{-1}$ using the points $1, \frac{4}{3}$, and 2.

2. Compute the approximate value of $\int_0^1 (x^2+1)^{-1} dx$ by using the composite trapezoid rule with three points. Then compare with the actual value of the integral.

3. If the composite trapezoid rule is used to compute $\int_{-1}^{2} \sin x \; dx$ with $h = 0.01$, give a realistic bound on the error.

4. How large must n be if the composite trapezoid rule is being used to estimate $\int_0^{\pi} \sin x \; dx$ with error $\leq 10^{-12}$? Will the estimate be too big or too small?

5. Consider $\int_1^2 dx/x^3$. What is the result of using the composite trapezoid rule with the partition points $1, \frac{3}{2}$, and 2?

6. Approximate $\int_0^2 2^x dx$ using the composite trapezoid rule with $h = \frac{1}{2}$.

7. We want to approximate $\int_1^2 f(x)\ dx$ given the table of values. Compute an estimate by the composite trapezoid rule. Can upper and lower sums be computed from the given data?

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
f(x)	10	8	7	6	5

8. Compute $\int_0^1 (1+x^2)^{-1} dx$ by the basic Simpson's Rule, using the three partition points $x = 0, 0.5$, and 1. Compare with the true solution.

9. Find an approximate value of $\int_1^2 x^{-}1\ dx$ using the basic Simpson's Rule with uniform spacing. Give a bound on the error.

10. Find a formula of the type
$$\int_0^1 f(x)\ dx \approx \alpha f(0) + \beta f(1)$$
that gives correct values for $f(x) = 1$ and $f(x) = x^2$. Does your formula give the correct value when $f(x) = x$?

11. Approximate
$$\int_0^2 e^{-x^2}\ dx$$
using the three point Gaussian Quadrature formula
$$\int_{-1}^1 f(x)\ dx \approx \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right).$$