Analytic Trigonometry

7.7 Inverse Trigonometric Functions

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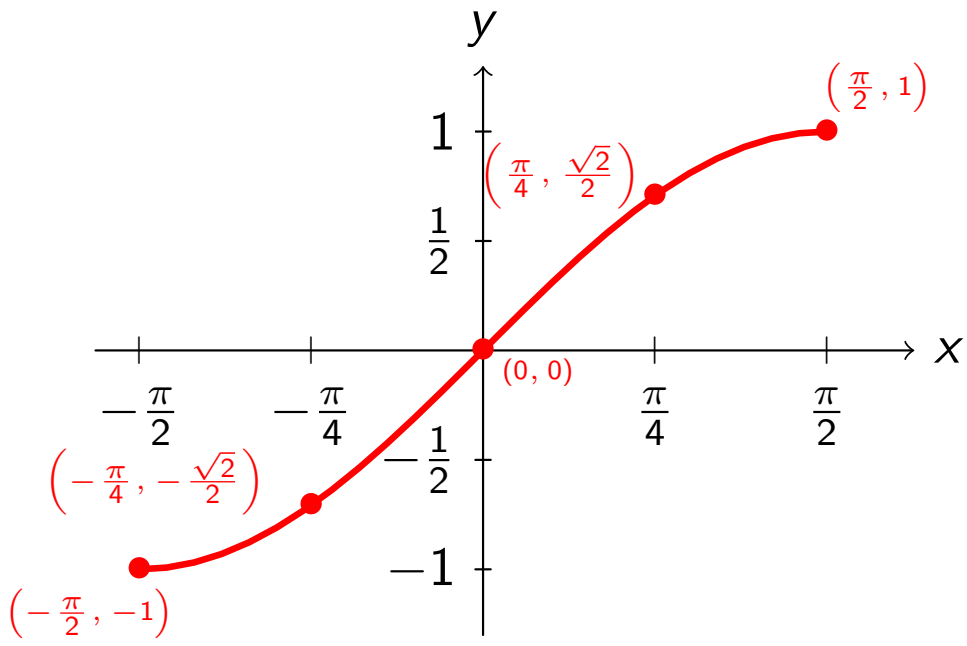
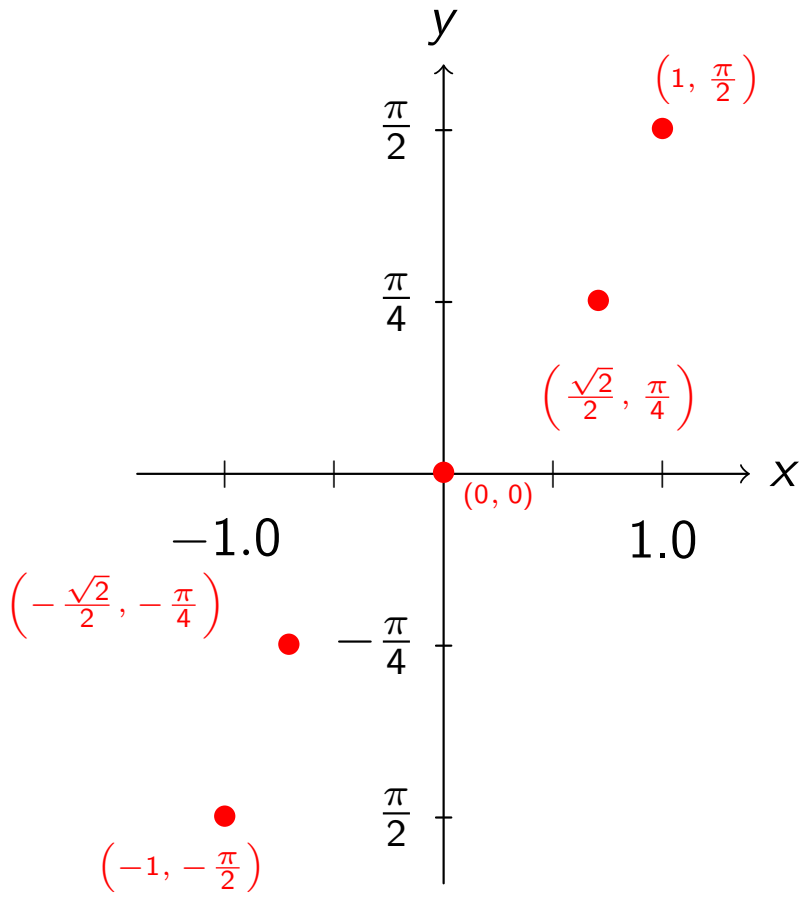
Start with the sine function with the restricted domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$y = \sin x$ Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Range:$[-1, 1]$



$y = \sin^{-1} x$ Domain:$[-1, 1]$ Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Inverse Sine Function

$$y = \sin^{-1} x \text{ or } y = \arcsin x \quad \text{means} \quad x = \sin y$$

where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Example

Find the exact value of each of the following expressions:

- $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
- $\arcsin\left(-\frac{1}{2}\right)$

Sine-inverse Sine Identities

$$\sin^{-1}(\sin x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\sin(\sin^{-1} x) = x \quad \text{for } -1 \leq x \leq 1$$

Example

Find the exact value of each of the following trigonometric expressions:

- $\sin\left[\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$
- $\sin^{-1}\left[\sin\left(\frac{3\pi}{4}\right)\right]$

Inverse cosine Function

$$y = \cos^{-1} x \text{ or } y = \arccos x \quad \text{means} \quad x = \cos y$$

where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$.

Example

Find the exact value of each of the following expressions:

- $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
- $\arccos 0$

Cosine-inverse Cosine Identities

$$\cos^{-1}(\cos x) = x \qquad \text{for } 0 \leq x \leq \pi$$

$$\cos(\cos^{-1} x) = x \qquad \text{for } -1 \leq x \leq 1$$

Example

Find the exact value of each of the following trigonometric expressions:

- $\cos\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$
- $\cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right]$

Inverse Tangent Function

$$y = \tan^{-1} x \text{ or } y = \arctan x \quad \text{means} \quad x = \tan y$$

where $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Example

Find the exact value of each of the following expressions:

- $\tan^{-1}\left(\sqrt{3}\right)$
- $\arctan 0$

Tangent-inverse Tangent Identities

$$\tan^{-1}(\tan x) = x \quad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\tan(\tan^{-1} x) = x \quad \text{for } -\infty \leq x \leq \infty$$

Example

Find the exact value of each of the following trigonometric expressions:

- $\tan(\tan^{-1} 17)$
- $\tan^{-1}\left(\tan \frac{2\pi}{3}\right)$

Example

Find the exact value of

$$\cos\left[\sin^{-1}\left(\frac{2}{3}\right)\right].$$