

# Analytic Trigonometry

## 7.5 Half-Angle Identities

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## Half-Angle Identities: Sine

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{2}}$$

## Half-Angle Identities: Cosine

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 + \cos A}{2}}$$

## Half-Angle Identities: Tangent

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

### Example

Use half-angle identity to find  $\sin 22.5^\circ$ .

### Example

Use half-angle identity to find  $\tan\left(\frac{\pi}{8}\right)$ .

### Example

If  $\cos x = -\frac{5}{13}$  and  $\sin x < 0$ , find  $\cos\left(\frac{x}{2}\right)$ .

## Example

Verify the identities

▶  $\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \cos x.$

▶  $\tan\left(\frac{A}{2}\right) + \cot\left(\frac{A}{2}\right) = 2 \csc A.$

## Example

$$\text{Graph } y = \frac{\sin(2\pi x)}{1 + \cos(2\pi x)}.$$

Simplify the trigonometric expression using a half-angle identity for the tangent function.

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A} \quad y = \frac{\sin(2\pi x)}{1 + \cos(2\pi x)}$$

Let  $A = 2\pi x$ . Then

$$\tan(\pi x) = \frac{\sin(2\pi x)}{1 + \cos(2\pi x)}$$

So we just graph  $y = \tan(\pi x)$ .