Analytic Trigonometry

7.4 Double–Angle Identities

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Double–Angle Identities: Sine

$$\sin(2A) = 2\sin A\cos A$$

Double–Angle Identities: Cosine

$$\begin{aligned}\cos(2A) &= \cos^2 A - \sin^2 A\\ \cos(2A) &= 1 - 2\sin^2 A\\ \cos(2A) &= 2\cos^2 A - 1\end{aligned}$$

Double–Angle Identities: Tangent

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

Example

If $\sin x = \frac{1}{\sqrt{5}}$ and $\cos x < 0$, find the exact value of $\sin(2x)$.

Example

If $\cos x = -\frac{5}{13}$ and $\sin x < 0$, find the exact value of

- $\cos(2x)$.
- $\tan(2x)$.

Example

Simplify each expression. Evaluate the resulting expression exactly, if possible.

- $\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$.
- $\frac{2\tan\left(\frac{5\pi}{12}\right)}{1-\tan^2\left(\frac{5\pi}{12}\right)}$.

Example

Verify the identities

- $(\sin x + \cos x)^2 = 1 + \sin(2x)$.
- $\sin^2 x = \frac{1 - cos(2x)}{2}$.