Analytic Trigonometry

7.3 Sum and Difference Identities

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Sum and Difference Identities for the Cosine Function

$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(A-B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

Example

Evaluate each of the following cosine expressions exactly.

1. $\cos\left(\frac{\pi}{12}\right)$
2. $\cos 195^\circ$

Example

Use the sum or the difference identity for the cosine function to write the following expressions as a single cosine expression.

$$\cos(4x)\cos(7x) + \sin(4x)\sin(7x)$$

Cofunction Identities for the Sine and Cosine Function

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Sum and Difference Identities for the Sine Function

$$\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \\ \sin(A-B) &= \sin A\cos B - \cos A\sin B \end{aligned}$$

Example

Use the sum or the difference identity for the sine function to evaluate each sine expression exactly.

1. $\sin\left(\dfrac{7\pi}{12}\right)$
2. $\sin 15^\circ$

Sum and Difference Identities for the Tangent Function

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Example

Find the exact value of $\tan\left(\frac{13\pi}{12}\right)$.

Example

Find the exact value of $\tan(\alpha - \beta)$ if $\sin\alpha = -\frac{3}{5}$ and $\cos\beta = -\frac{1}{4}$ and the terminal side of α lies in *QIII* and the terminal side of β lies in *QII*.