Analytic Trigonometric

7.1 Basic Trigonometric Identities

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Reciprocal Identites	Equivalent Forms	Domain Restrictions
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin\theta = \dfrac{1}{\csc\theta}$	$\theta \neq n\pi \quad n = \text{integer}$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$	$\theta \neq \frac{n\pi}{2} \quad n = \text{odd integer}$
$\cot\theta = \dfrac{1}{\tan\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$	$\theta \neq \frac{n\pi}{2} \quad n = \text{integer}$

Example

Use a reciprocal identity to find the function value indicated.

(a) If $\sin \theta = -\frac{3}{7}$, find $\csc \theta$.

(b) If $\cos \theta = 0.8$, find $\sec \theta$.

(c) If $\tan \theta = 0.5$, find $\cot \theta$.

(d) If $\sec \theta = \frac{\sqrt{11}}{2}$, find $\cos \theta$.

(e) If $\cot \theta = 3.5$, find $\tan \theta$.

Quotient Identities

Quotient Identites	Domain Restrictions
$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$	$\cos \theta \neq 0$ or $\theta \neq \frac{n\pi}{2}$ $n = $ odd integer
$\cot \theta = \dfrac{\cos \theta}{\sin \theta}$	$\sin \theta \neq 0$ or $\theta \neq n\pi$ $n = $ integer

Example

Use a quotient identity to find the function value indicated.

(a) If $\sin \theta = -\frac{1}{2}$ and $\cos \theta = \frac{\sqrt{3}}{2}$, find $\cot \theta$.

(b) If $\sin \theta = -0.6$ and $\cos \theta = -0.8$, find $\tan \theta$.

(c) If $\sin \theta = -\frac{\sqrt{11}}{6}$ and $\cos \theta = -\frac{5}{6}$, find $\cot \theta$.

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

Example

Use a Pythagorean identity to find the function value indicated.

(a) If $\sin \theta = -\frac{3}{5}$ and the terminal side of θ lies in quadrant III, find $\cos \theta$.

(b) If $\cos \theta = \frac{2}{7}$ and the terminal side of θ lies in quadrant IV, find $\sin \theta$.

(c) If $\tan \theta = -5$ and the terminal side of θ lies in quadrant II, find $\sec \theta$.

Example

Use identities to find the function value indicated.

(a) Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = -\frac{4}{3}$ and the terminal side of θ lies in quadrant II.

(b) Find $\sin\theta$ and $\cos\theta$ if $\cot\theta = 0.1$ and the terminal side of θ lies in quadrant III.

Example

Perform the indicated operation and simplify your answers, if possible. Leave all answers in terms of $\sin\theta$ and $\cos\theta$.

(a) $\sec\theta\cot\theta$

(b) $\tan^2\theta - \sec^2\theta$

(c) $\csc\theta - \sin\theta$