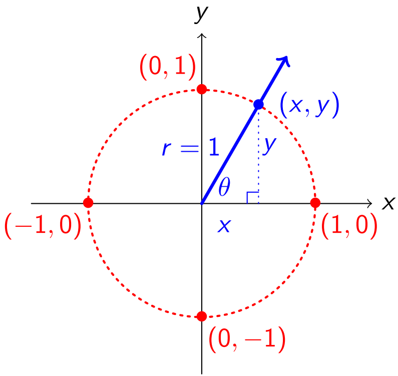
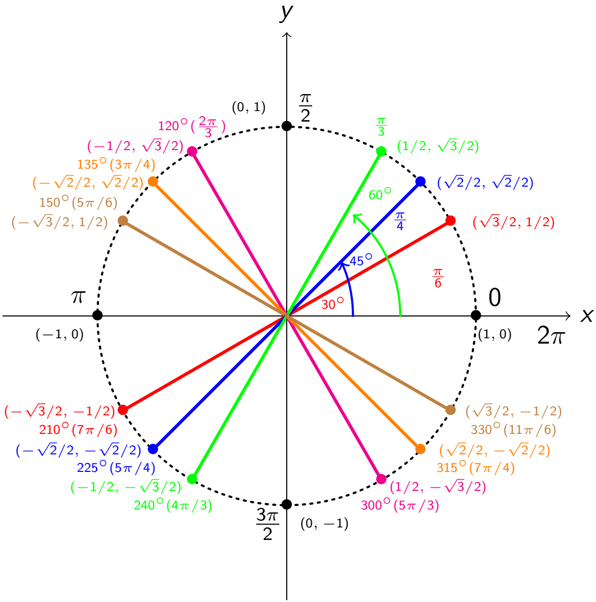
Trigonometric Functions

6.7 Definition 3 of Trigonometric Functions: Unit Circle

November 2, 2010



The point (x, y) on the unit circle can be written as $(\cos\theta, \sin\theta)$.


y
x
$\frac{\pi}{2}$
(0, 1)
$120^\circ (\frac{2\pi}{3})$
$(-1/2, \sqrt{3}/2)$
$135^\circ (3\pi/4)$
$(-\sqrt{2}/2, \sqrt{2}/2)$
$150^\circ (5\pi/6)$
$(-\sqrt{3}/2, 1/2)$
$\frac{\pi}{3}$
$(1/2, \sqrt{3}/2)$
$(\sqrt{2}/2, \sqrt{2}/2)$
60°
$\frac{\pi}{4}$
$(\sqrt{3}/2, 1/2)$
45°
$\frac{\pi}{6}$
30°
π
0
(−1, 0)
(1, 0)
2π
$(-\sqrt{3}/2, -1/2)$
$210^\circ (7\pi/6)$
$(-\sqrt{2}/2, -\sqrt{2}/2)$
$225^\circ (5\pi/4)$
$(-1/2, -\sqrt{3}/2)$
$240^\circ (4\pi/3)$
$\frac{3\pi}{2}$
(0, −1)
$(\sqrt{3}/2, -1/2)$
$330^\circ (11\pi/6)$
$(\sqrt{2}/2, -\sqrt{2}/2)$
$315^\circ (7\pi/4)$
$(1/2, -\sqrt{3}/2)$
$300^\circ (5\pi/3)$

Definition 3: Trigonometric Functions

Unit Circle Approach

Let (x, y) be any point on the unit circle. If θ is any real number that represents the distance from the point $(1, 0)$ along the circumference to the point (x, y), then

$$\sin\theta = y \qquad \cos\theta = x \qquad \tan\theta = \frac{y}{x} \quad x \neq 0$$

$$\csc\theta = \frac{1}{y} \quad y \neq 0 \qquad \sec\theta = \frac{1}{x} \quad x \neq 0 \qquad \cot\theta = \frac{x}{y} \quad y \neq 0$$

Example

Find the exact value for

(a) $\sin\left(\frac{5\pi}{3}\right)$

(b) $\cos\left(\frac{7\pi}{6}\right)$

(c) $\tan\left(\frac{7\pi}{4}\right)$

Example

Use the unit circle to find all values of θ, $0 \leq \theta \leq 2\pi$, for which $\cos\theta = -\frac{\sqrt{3}}{2}$.

Domains and Ranges of the Circular Functions

For any real number θ and integer n:

FUNCTION	**DOMAIN**	**RANGE**
$\sin\theta$	$(-\infty, \infty)$	$[-1, 1]$
$\cos\theta$	$(-\infty, \infty)$	$[-1, 1]$
$\tan\theta$	all real numbers such that $\theta \neq \frac{(2n+1)\pi}{2}$	$(-\infty, \infty)$
$\cot\theta$	all real numbers such that $\theta \neq n\pi$	$(-\infty, \infty)$
$\sec\theta$	all real numbers such that $\theta \neq \frac{(2n+1)\pi}{2}$	$(-\infty, -1] \cup [1, \infty)$
$\csc\theta$	all real numbers such that $\theta \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$

Even and Odd Functions

The cosine function is an even function

$$\cos\theta = \cos(-\theta)$$

The sine function is an odd function

$$\sin(-\theta) = -\sin(\theta)$$

Example

Evaluate $\sin\left(-\frac{7\pi}{6}\right)$.

Example

Use a calculator to evaluate $\sin\left(\frac{5\pi}{12}\right)$. Round the answer to four decimal places.

Example

Show that the cosecant function is an odd function.