Trigonometric Functions

6.5 Trigonometric Functions of Nonacute Angles

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Algebraic Signs of Trigonometric Functions

PHRASE	QUADRANT	POSITIVE TRIGONOMETRIC FUNCTION
All	I	**A**ll three: sine, cosine and tangent
Students	II	**S**ine
Take	III	**T**angent
Calculus	IV	**C**osine

Example

If $\tan\theta = -\frac{1}{2}$ and the terminal side of θ lies in quadrant II, find $\cos\theta$.

Ranges of the Trigonometric Functions

For any angle θ for which the trigonometric functions are defined, the six trigonometric functions have the following ranges:

- $-1 \leq \sin\theta \leq 1$
- $-1 \leq \cos\theta \leq 1$
- $\sec\theta \leq -1$ or $\sec\theta \geq 1$
- $\csc\theta \leq -1$ or $\csc\theta \geq 1$
- $\tan\theta$ and $\cot\theta$ can equal any real number

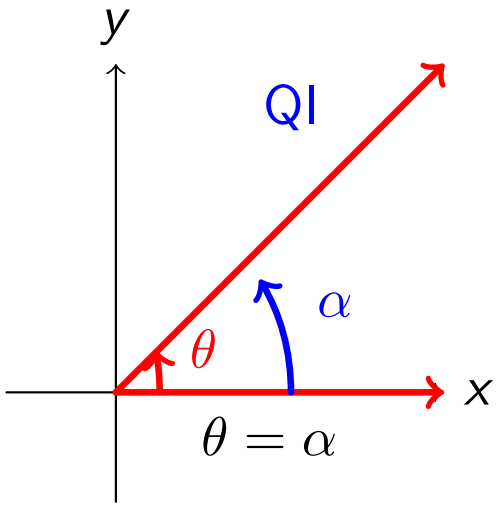
Example

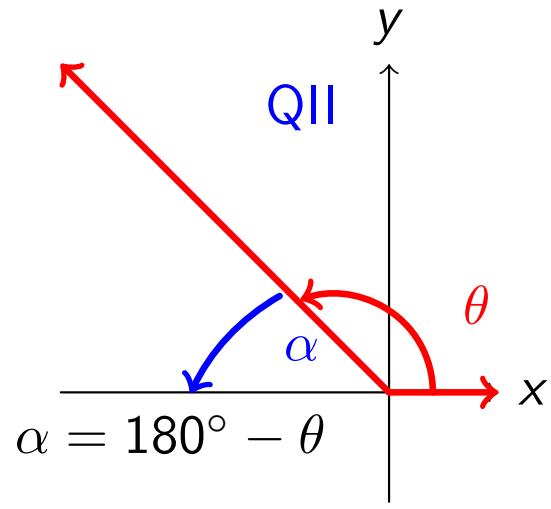
Determine whether each statement is possible or not.

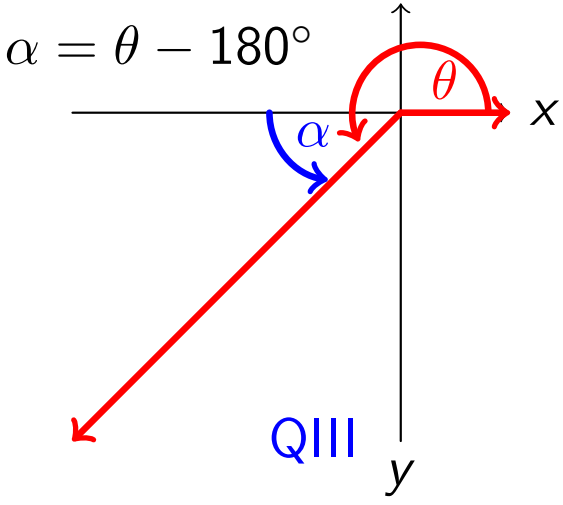
(a) $\cos\theta = 1.0002$ **(b)** $\cot\theta = 0$ **(c)** $\sec\theta = \dfrac{\sqrt{2}}{2}$

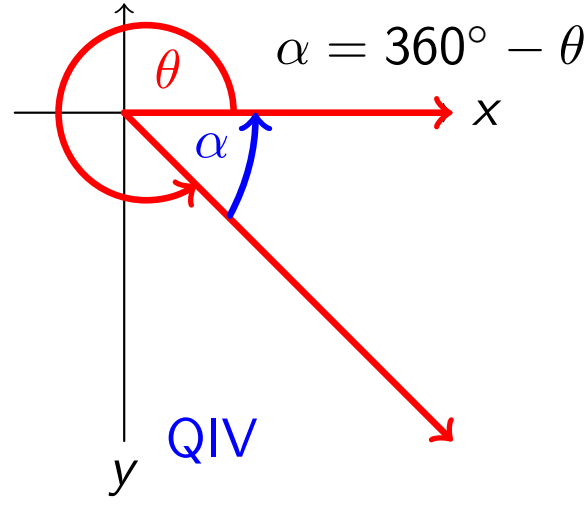
Definition: Reference Angle

For angle θ, $0^\circ < \theta < 360^\circ$, in standard position whose terminal side lies in one of the four quadrants, there exists a **reference angle** α that is the acute angle with positive measure formed by the terminal side of θ and the x-axis.


y
QI
α
θ
x
θ = α


y
QII
θ
α
x
α = 180° − θ


α = θ − 180°
θ
α
x
QIII
y


α = 360° − θ
θ
x
α
y
QIV

Example

Find the reference angle for each angle given.

(a) 210°

(b) 135°

(c) 422°

Definition: Reference Right Triangle

To form a **reference right triangle** for angle θ, where $0° < \theta < 360°$, drop a perpendicular line from the terminal side of the angle to the x-axis. The right triangle now has reference angle α as one of its angles.

Example

Find the exact value of

(a) $\cos 120^\circ$
(b) $\tan 210^\circ$
(c) $sec(-330^\circ)$

Example

Find all possible values of θ, where $0^\circ < \theta < 360^\circ$ when

(a) $\sin\theta = \frac{\sqrt{3}}{2}$
(b) $\cos\theta = -1$
(c) $\sin\theta = 0$