

Exponential and Logarithmic Functions

5.4 Exponential and Logarithmic Equations

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One-to-One Properties

$$b^x = b^y \quad \text{if and only if} \quad x = y$$

$$\log_b x = \log_b y \quad \text{if and only if} \quad x = y$$

Inverse Properties

$$b^{\log_b x} = x \quad x > 0$$

$$\log_b b^x = x$$

Examples

Solve the exponential equations using the one-to-one property.

a. $3^x = 81$ **b.** $5^{7-x} = 125$ **c.** $(\frac{1}{2})^{4x} = 16$

Examples

Solve the exponential equations and round the answers to four decimal places.

a. $5^{3x} = 16$ **b.** $4^{3x+2} = 71$

Example

Solve the exponential equation

$$4e^{x^2} = 64$$

and round the answer to four decimal places. $x \approx \pm 1.6651$

Example

Solve the exponential equation

$$e^{2x} - 4e^x + 3 = 0$$

and round the answer to four decimal places. $x \approx 1.0986$ or $x = 0$.

Example

Solve the equation

$$\log_4(2x - 3) = \log_4(x) + \log_4(x - 2)$$

$$x = 4.$$

Example

Solve the equation

$$\log_3(9x) - \log_3(x - 8) = 4$$

$$x = 9.$$

Example

Solve the equation

$$\ln(3 - x^2) = 7$$

no real solution.

Example

You save \$1,000 from a summer job and put it in a CD earning 5% compounding continuously. How many years will it take for your money to double? Round to the nearest year.