Exponential and Logarithmic Functions

5.3 Properties of Logarithms

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Properties of Exponents

Let a, b, m and n be any real numbers and $m > 0$ and $n > 0$, then the following are true:

1. $b^m \cdot b^n = b^{m+n}$
2. $b^{-m} = \dfrac{1}{b^m} = \left(\dfrac{1}{b}\right)^m$
3. $\dfrac{b^m}{b^n} = b^{m-n}$
4. $(b^m)^n = b^{mn}$
5. $(ab)^m = a^m \cdot b^m$
6. $b^0 = 1, \quad b \neq 0$
7. $b^1 = b$

Properties of Logarithms

If b, M, and N are positive real numbers, where $b \neq 1$ and p and x are real numbers, then the following are true:

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x, \quad x > 0$
5. $\log_b MN = \log_b M + \log_b N$ Product Rule
6. $\log_b \left(\dfrac{M}{N} \right) = \log_b M - \log_b N$ Quotient Rule
7. $\log_b M^p = p \log_b M$ Power Rule

Examples

Use properties of logarithms to simplify the expressions:

(a). $\log_{10} 10$ **(b).** $\ln 1$ **(c).** $10^{\log(x+8)}$

(d). $e^{\ln(2x+5)}$ **(e).** $\log 10^{x^2}$ **(f).** $\ln e^{x+3}$

Example

Use the logarithmic properties to write the expression $\log_b(u^2\sqrt{v})$ as a sum of simpler logarithms.

Example

Write the expression $\ln\left(\dfrac{x^3}{y^2}\right)$ as a difference of logarithms.

Example

Write $\ln\left(\dfrac{x^2 - x - 6}{x^2 + 7x + 6}\right)$ as a sum or difference of logarithms.

Example

Write the expression

$$\frac{2}{3} \ln x - \frac{1}{2} \ln y$$

as a logarithm of a quotient.

Example

Write the expression

$$\frac{1}{2} \log_b x + \log_b(2x + 1) - 2 \log_b 4$$

as a single logarithm.

Change-of-Base Formula

For any logarithmic bases a and b and any positive number M, the change-of-base formula says that

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

Example

Use the change-of-base formula to evaluate $\log_4 17$. Round to four decimal places.