Exponential and Logarithmic Functions

5.2 Logarithmic Functions and Their Graphs

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Definition: Logarithmic Function

For $x > 0$, $b > 0$ and $b \neq 1$, the **logarithmic function** with **base** b is denoted

$$f(x) = \log_b x$$

where

$$y = \log_b x \text{ if and only if } x = b^y.$$

Examples

Write each equation in its equivalent exponential form.

(a). $\log_2 8 = 3$ **(b).** $\log_9 3 = \frac{1}{2}$ **(c).** $\log_5 \left(\frac{1}{25}\right) = -2$

Examples

Write each equation in its equivalent logarithmic form.

(a). $16 = 2^4$ **(b).** $9 = \sqrt{81}$ **(c).** $\frac{1}{9} = 3^{-2}$ **(d).** $x^a = z$

Examples

Find the exact value of

(a). $\log_3 81$ **(b).** $\log_{169} 13$ **(c).** $\log_5 \left(\frac{1}{5}\right)$

- The logarithmic function of base 10 is called the **common logarithmic function**.
  - If no explicit base is indicated, base 10 is implied.
- The logarithmic function of base e is called the **natural logarithmic function**.
  - The natural logarithmic function $f(x) = \log_e x$ is often expressed as $f(x) = \ln x$.

Examples

Use a calculator to evaluate the common and natural logarithms. Round your answers to four decimal places.

(a). $\log 415$ **(b).** $\ln 415$ **(c).** $\log 1$ **(d).** $\ln 1$ **(e).** $\log(-2)$

The general logarithmic function $y = \log_b x$ is defined as the inverse of the exponential function $y = b^x$.

Comparison of Inverse Functions: $f(x) = \log_b x$ and $f^{-1}(x) = b^x$

EXPONENTIAL FUNCTION	**LOGARITHMIC FUNCTION**
$y = b^x$	$y = \log_b x$
y-intercepts: $(0, 1)$	x-intercepts: $(1, 0)$
Domain $(-\infty, \infty)$	Domain $(0, \infty)$
Range $(0, \infty)$	Range $(-\infty, \infty)$
Horizontal asymptote: x-axis	Vertical asymptote: y-axis

Examples

Find the domain of each of the given logarithmic functions.

(a). $f(x) = \log_b(x-4)$ **(b).** $g(x) = \log_b(5-2x)$

Examples

Find the domain of each of the given logarithmic functions.

(a). $\ln(x^2-9)$ **(b).** $\log(|x+1|)$

Examples

Graph the functions, and state the domain and range of each.

$$\textbf{(a).}\ y = \log_2(x-3) \quad \textbf{(b).}\ y = \log_2(x) - 3$$

Example

Graph the function, and state its domain and range.

$$f(x) = -\log_2(x-3)$$