Polynomial and Rational Functions

4.1 Quadratic Functions

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Definition: Polynomial Function

Let n be a nonnegative integer, and let $a_n, a_{n-1}, ..., a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$$

is called a **polynomial function of** x **with degree** n. The coefficient a_n is called the **leading coefficient**, and a_0 is the constant.

Examples

Polynomial	**Degree**	**Special Name**
$f(x) = 10$	0	Constant function
$f(x) = -2x + 1$	1	Linear function
$f(x) = 7x^2 - 5x + 19$	2	Quadratic function
$f(x) = 4x^3 + 2x - 7$	3	Cubic function

Graph of a Quadratic Function

Definition: Quadratic Function

Let a, b, and c be real numbers with $a \neq 0$. The function

$$f(x) = ax^2 + bx + c$$

is called a **quadratic function**.

The graph of any quadratic function is a **parabola**.

- If the leading coefficient a is *positive*, then the parabola opens *up*.
- If the leading coefficient a is *negative*, then the parabola opens *down*.

The **vertex** (or turning point)

- is the minimum point, or low point, on the graph if the parabola opens up.
- is the maximum point or high point, on the graph if the parabola opens down.

The vertical line that intersects the parabola at the vertex is called the **axis of symmetry**.

- The axis of symmetry is the line $x = h$.
- The vertex is located at the point (h, k).

Graphing Quadratic Functions in Standard Form

Quadratic Function: Standard Form

The quadratic function

$$f(x) = a(x - h)^2 + k$$

is in **standard form**.

- The graph of f is a parabola whose vertex is the point (h, k).
- The parabola is symmetric with respect to the line $x = h$.
- If $a > 0$, the parabola opens up.
- If $a < 0$, the parabola opens down.

Graphing Quadratic Functions

To graph $f(x) = a(x - h)^2 + k$.

- **Step 1:** Determine whether the parabola opens up or down.
  - $a > 0$ up
  - $a < 0$ down
- **Step 2:** Determine the vertex (h, k).
- **Step 3:** Find the y-intercept.
- **Step 4:** Find any x-intercepts.
- **Step 5:** Plot the vertex and intercepts and connect them with a smooth curve.

Example 1

Graph the quadratic function $f(x) = (x-3)^2 - 1$.

Example 2

Graph the quadratic function $f(x) = -2(x-1)^2 - 3$.

Example 3

Graph the quadratic function $f(x) = x^2 - 6x + 4$.

Graphing Quadratic Functions in General Form

Vertex of a parabola

The graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with the **vertex** located at the point

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

Graphing a Quadratic Function in General Form

- **Step 1:** Find the vertex
- **Step 2:** Determine whether the parabola opens up or down.
  - If $a > 0$, the parabola opens up.
  - If $a < 0$, the parabola opens down.
- **Step 3:** Find additional points near the vertex.
- **Step 4:** Sketch the graph with a parabolic curve.

Example 5

Sketch the graph of $f(x) = -2x^2 + 4x + 5$.

Finding the Equation of a Parabola

Example 6

Find the equation of a parabola whose graph has a vertex at $(3, 4)$ and which passes through the point $(2, 3)$. Express the quadratic function in both standard and general forms.

Example 7

A company that produces motorcycles has a daily production cost of

$$C(x) = 2000 - 15x + 0.05x^2$$

where C is the cost in dollars to manufacture a motorcycle and x is the number of motorcycles produced. How many motorcycles can be produced each day in order to minimize the cost of each motorcycle? What is the corresponding minimum cost?