Functions and their Graphs

3.4 Operations on Functions and Composition of Functions

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Adding, Subtracting, Multiplying and Dividing Functions

Function	**Notation**	**Domain**
Sum	$(f+g)(x) = f(x) + g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Difference	$(f-g)(x) = f(x) - g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\{\text{domain of } f\} \cap \{\text{domain of } g\}$ $\cap \{\ g(x) \neq 0\}$

Example 1

For the functions $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{4-x}$, determine the sum function, difference function, product function and quotient function. State the domain of the four new functions.

Example 2

Given the functions $F(x) = \sqrt{x}$ and $G(x) = |x - 3|$, find the quotient function, $\left(\frac{F}{G}\right)(x)$, and state its domain.

Composition of Function

Notation	Words	Definition	Domain
$f \circ g$	f composed with g	$f(g(x))$	The set of all real numbers x in the domain of g such that $g(x)$ is also in the domain of f.
$g \circ f$	g composed with f	$g(f(x))$	The set of all real numbers x in the domain of f such that $f(x)$ is also in the domain of g.

Example 3

Given the functions $f(x) = x^2 + 1$ and $g(x) = x - 3$, find $(f \circ g)(x)$.

Example 4

Given the functions $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x}$, determine $f \circ g$, and state its domain.

Example 6

Given the functions $f(x) = x^2 - 7$ and $g(x) = 5 - x^2$, evaluate

- $f(g(1))$
- $f(g(-2))$
- $g(f(3))$
- $g(f(-4)$