Functions and their Graphs

3.3 Graphing Techniques: Transformations

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Vertical Shifts

Assuming that c is a positive constant

To Graph	**Shift the Graph of** f(x)
$f(x) + c$	c units upward
$f(x) - c$	c units downward

Adding or subtracting a constant **outside** the function corresponds to a **vertical** shift that goes **with the sign**.

Horizontal Shifts

Assuming that c is a positive constant

To Graph	**Shift the Graph of** f(x)
$f(x + c)$	c units to the left
$f(x - c)$	c units to the right

Adding or subtracting a constant **inside** the function corresponds to a **horizontal** shift that goes **opposite the sign**.

Example 1

Sketch the graphs of the given functions using horizontal and vertical shifts.

(a) $g(x) = x^2 - 1$
(b) $H(x) = (x + 1)^2$

Example 2

Graph the functions using translations and state the domain and range of each function.

(a) $g(x) = \sqrt{x + 1}$
(b) $H(x) = \sqrt{x} - 2$

Example 3

Sketch the graph of the function $F(x) = (x + 1)^2 - 2$. State the domain and range of F.

Reflection About the Axes

The graph of $-f(x)$ is obtained by reflecting the graph of $f(x)$ about the x-axis.

The graph of $f(-x)$ is obtained by reflecting the graph of $f(x)$ about the y-axis.

Example 4

Sketch the graph of the function $G(x) = -\sqrt{x+1}$.

Example 5

Sketch the graph of the function $f(x) = \sqrt{2-x} + 1$.

Vertical stretching and vertical compressing of graphs

The graph of $cf(x)$ is found by:

- **Vertically stretching** the graph of $f(x)$ if $c > 1$
- **Vertically compressing** the graph of $f(x)$ if $0 < c < 1$

Note: c is any positive real number.

Example 6

Graph the function $h(x) = \frac{1}{4}x^3$.

Horizontal stretching and horizontal compressing of graphs

The graph of $f(cx)$ is found by:

- **Horizontally stretching** the graph of $f(x)$ if $0 < c < 1$
- **Horizontally compressing** the graph of $f(x)$ if $c > 1$

Note: c is any positive real number.

Example 7

Given the graph of $f(x)$, graph

(a) $2f(x)$

(b) $f(2x)$