Functions and their Graphs

3.2 Graphs of Functions; Piecewise-Defined Functions; Increasing and Decreasing Functions; Average Rate of Change

September 16, 2010

Linear Function: $f(x) = mx + b$, m and b are real numbers

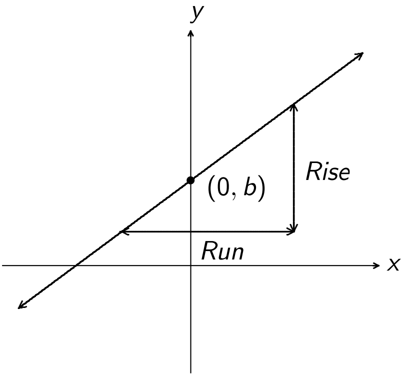


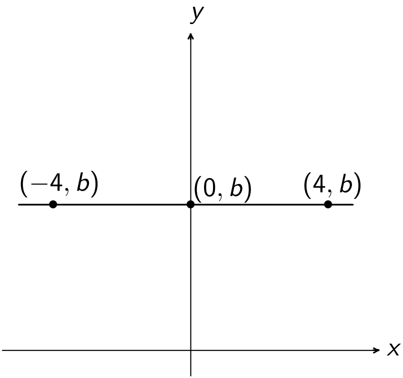
Figure: $m \neq 0$

The domain is the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

The range is the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

The graph has slope m and y-intercept b.

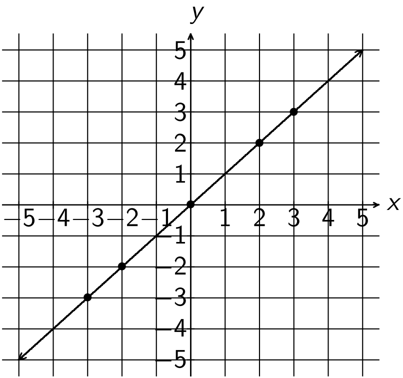
Constant Function: $f(x) = b$ $\quad$ b is any real number



Domain is the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

The range is a single value b.

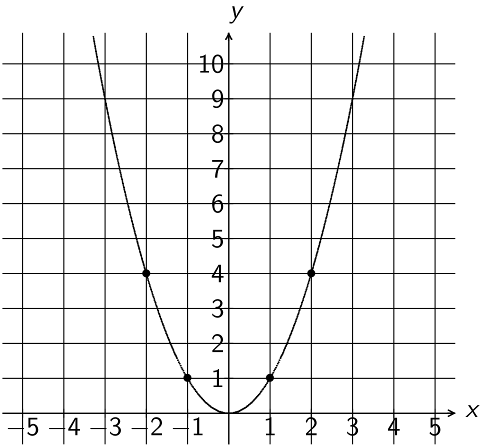
Identity Function: $f(x) = x$



Both the domain and range are the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

It passes through the origin.

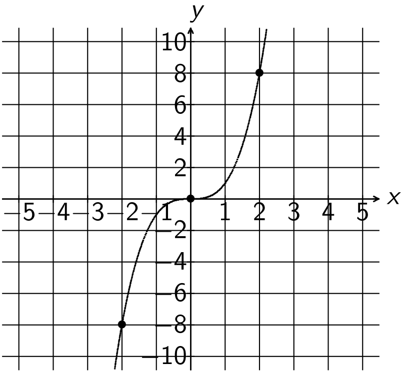
Square Function: $f(x) = x^2$



The domain is the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

The range is the set of all nonnegative numbers. $[0, \infty)$

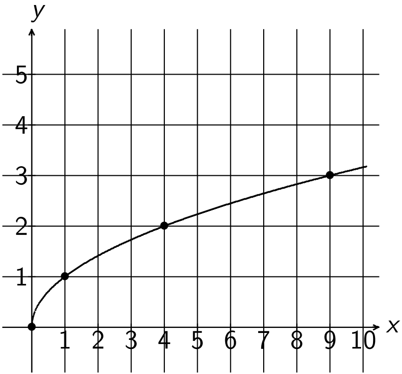
Cube Function: $f(x) = x^3$



The domain is the set of all real numbers $\mathbb{R}$. $(-\infty, \infty)$

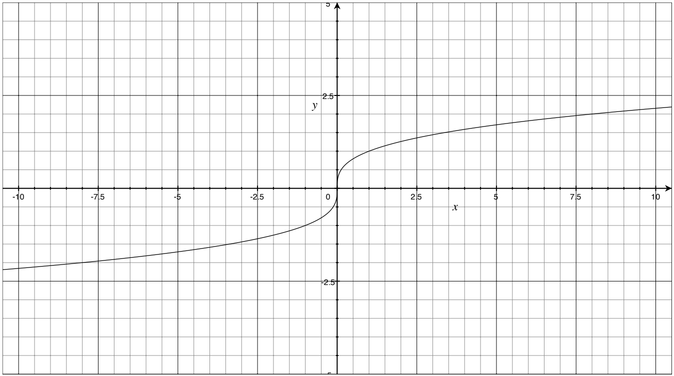
The range is the set of all real numbers.$\mathbb{R}$. $(-\infty, \infty)$

Square Root Function: $f(x) = \sqrt{x}$ or $f(x) = x^{\frac{1}{2}}$



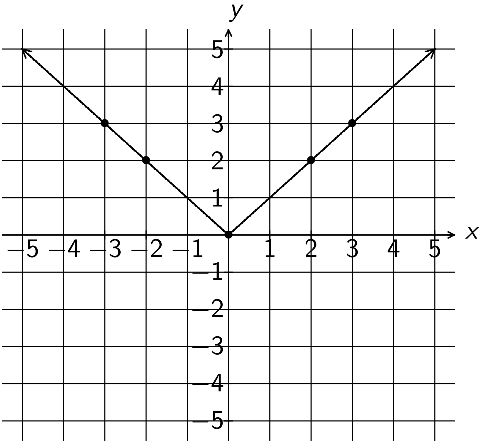
Domain: $[0, \infty)$ Range: $[0, \infty)$

Cube Root Function: $f(x) = \sqrt[3]{x}$ or $f(x) = x^{\frac{1}{3}}$



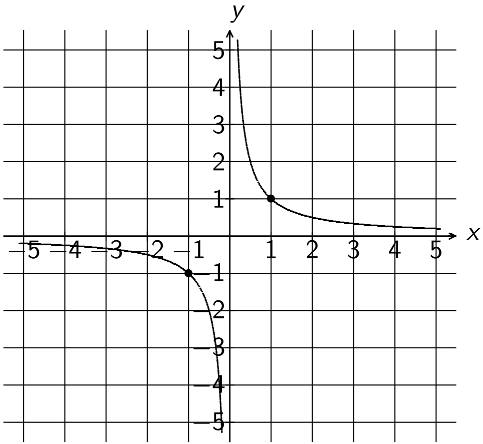
Doman: $(-\infty, \infty)$ Range: $(-\infty, \infty)$

Absolute Value Function: $f(x) = |x|$



Doman: $(-\infty, \infty)$ Range: $[0, \infty)$

Reciprocal Function: $f(x) = \dfrac{1}{x} \quad x \neq 0$



Doman: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$

Even and Odd Functions

Function	**Symmetric with Respect to**	**On Replacing x with $-x$**
Even	y-axis or vertical axis	$f(-x) = f(x)$
Odd	origin	$f(-x) = -f(x)$

Example 1

Determine whether the functions are even, odd or neither.

(a) $f(x) = x^2 - 3$

(b) $g(x) = x^5 + x^3$

(c) $h(x) = x^2 - x$

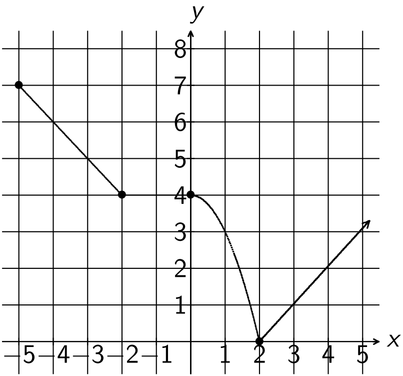
(d) $G(t) = |t + 2|$

(e) $F(x) = \sqrt{x^2 + 2}$

Increasing, Decreasing and Constant Functions

1. A function f is **increasing** on an open interval I if for any x_1 and x_2 in I, where $x_1 < x_2$, then $f(x_1) < f(x_2)$.
2. A function f is **decreasing** on an open interval I if for any x_1 and x_2 in I, where $x_1 < x_2$, then $f(x_1) > f(x_2)$.
3. A function f is **constant** on an open interval I if for any x_1 and x_2 in I, then $f(x_1) = f(x_2)$.

Example 2: Given the graph of the function



- State the domain and range of the function.
- Find the intervals when the function is increasing, decreasing or constant.

Average Rate of Change

Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be two distinct points, $(x_1 \neq x_2)$, on the graph of the function f. The **average rate of change** of f between x_1 and x_2 is given by

$$\textbf{Average Rate of Change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

Example 3

Find the average rate of change of $f(x) = x^4$ from

(a) $x = -1$ to $x = 0$

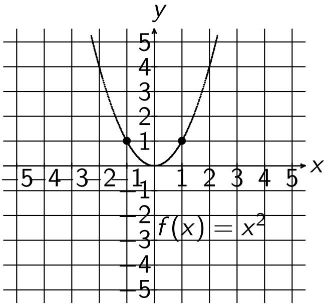
(b) $x = 0$ to $x = 1$

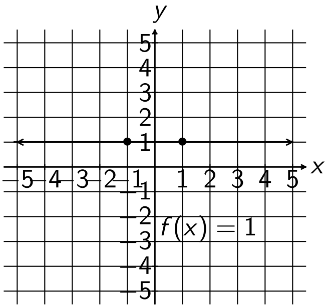
(c) $x = 1$ to $x = 2$

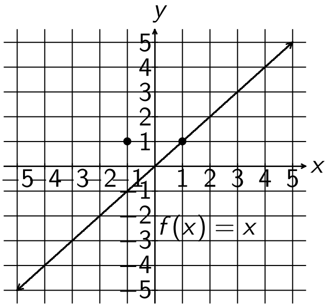
Example 5

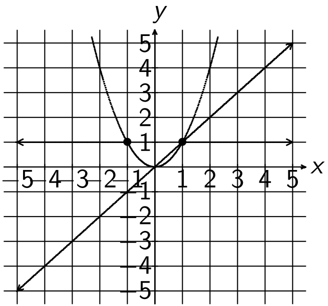
Graph the piecewise-defined function and state the domain, range, and intervals when the function is increasing, decreasing or constant.

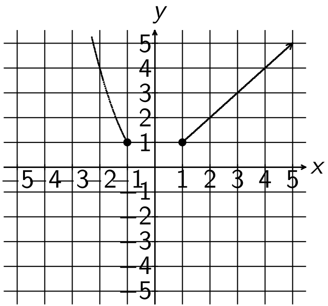
$$G(x) = \begin{cases} x^2 & x < -1 \\ 1 & -1 \leq x \leq 1 \\ x & x > 1 \end{cases}$$




y
x
5
4
3
2
1
−1
−2
−3
−4
−5
f(x) = 1


y
x
5
4
3
2
1
−1
−2
−3
−4
−5
f(x) = x





$G(x)$ is the last graph