Equations and Inequalities

1.7 Absolute Value Equations and Inequalities

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Definition

The **absolute value** of a real number a, denoted by the symbol $|a|$, is defined by

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

The absolute value of a real number is never negative.

Properties of Absolute Value

For all real numbers a and b,

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $|\frac{a}{b}| = \frac{|a|}{|b|}, \quad b \neq 0$

Distance between two points on the real number line

If a and b are real numbers, the **distance between** a **and** b is the absolute value of their difference given by

$$|a-b| \quad \text{or} \quad |b-a|.$$

Example (1)

Find the distance between -4 and 3 on the real number line.

Solution: $|-4-3| = |-7| = 7.$

Definition

Absolute Value Equation

If $|x| = a$, then $x = -a$ or $x = a$, where $a \geq 0$.

Example (2)

Solve the equation $|x - 3| = 8$ algebraically and graphically.

Graphical Interpretation: What numbers are 8 units away from 3 on the number line?

The solution set is $\{-5, 11\}$.

Example (3)

Solve the equation $|1 - 3x| = 7$.

The solution set is $\{-2, \frac{8}{3}\}$.

Example (5)

Solve the equation $|1-3x|=-7$.

No solution.

Example (6)

Solve the equation $|5-x^2|=1$.

The solution set is $\{\pm 2, \pm\sqrt{6}\}$

Properties of Absolute Value Inequalities

1. $|x| < a$ is equivalent to $-a < x < a$
2. $|x| \leq a$ is equivalent to $-a \leq x \leq a$
3. $|x| > a$ is equivalent to $x < -a$ or $x > a$
4. $|x| \geq a$ is equivalent to $x \leq -a$ or $x \geq a$

Example (7)

Solve the inequality $|3x - 2| \leq 7$.

The solution in interval notation is $[-\frac{5}{3}, 3]$.

Example (8)

Solve the inequality $|1 - 2x| > 5$.

The solution is $(-\infty, -2) \cup (3, \infty)$