Equations and Inequalities

1.6 Polynomial And Rational Inequalities

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Definition

Zeros of a polynomial are the values of x that make the polynomial equal to zero. These zeros divide the real number line into **test intervals** where the value of the polynomial is either positive or negative.

For example, consider the polynomial $x^2 + x - 2$. Its zeros are

$$\begin{aligned} x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \\ x = -2 \text{ or } x &= 1 \end{aligned}$$

Thus the zeros are $x = -2$ and $x = 1$. These zeros divide the real number line into three test intervals:

$$(-\infty, -2) \qquad (-2, 1) \qquad (1, \infty)$$

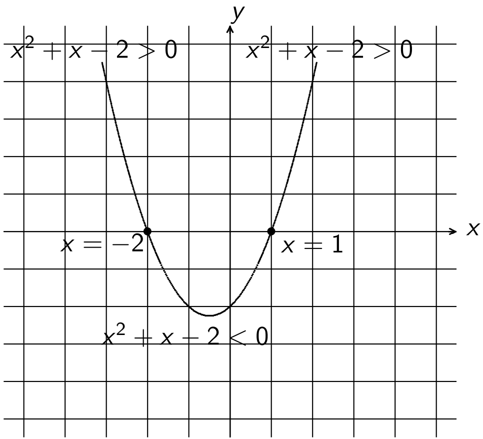

Figure: Graph of the polynomial $x^2 + x - 2$

Procedure for Solving Polynomial Inequalities

- **Step 1:** Write the inequality in *standard form*.
- **Step 2:** Identify zeros.
- **Step 3:** Draw the number line with zeros labeled.
- **Step 4:** Determine the sign of the polynomial in each interval.
- **Step 5:** Identify which interval(s) make the inequality true.
- **Step 6:** Write the solution in interval notation.

Example (1)

Solve the inequality $x^2 - x > 12$.

The solution is $(-\infty, -3)(4, \infty)$.

Example (2)

Solve the inequality $x^2 \leq 4$.

The solution is $[-2, 2]$.

Example (3)

Solve the inequality $x^2 + 2x \geq -3$.

The solution is $(-\infty, \infty)$.

In rational inequalities once expressions are combined into a single fraction, any values that make *either* the numerator *or* the denominator equal to zero divide the number line into intervals.

Example (7)

Solve the inequality

$$\frac{x-3}{x^2-4} \geq 0.$$

The solution is $(-2, 2) \cup [3, \infty)$.

Example (8)

Solve the inequality

$$\frac{x}{x+2} \leq 3.$$

The solution is $(-\infty, -3] \cup (-2, \infty)$.