# Equations and Inequalities

## 1.4 Other Types of Equations

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## Definition

**Radical Equations** are equations in which the variable is inside a radical.

Examples:

$$\sqrt{x-3} = 2 \qquad \sqrt{2x+3} = x \qquad \sqrt{x+2} + \sqrt{7x+2} = 6$$

A radical equation may be transformed into a simple linear or quadratic equations. Sometimes the transformation process yields **extraneous solutions**. These are apparent solutions that may solve the transformed problem but are not solutions of the original radical equation.

## Example (1)

Solve the equation $\sqrt{x-3}=2$.

Transform the radical equation into a linear equation...

The solution set is $\{7\}$.

## Example (2)

Solve the equation $\sqrt{2x+3}=x$.

Transform the radical equation into a quadratic equation...

The solution set is $\{3\}$.

## Example (3)

Solve the equation $\sqrt{x+2} + \sqrt{7x+2} = 6$.

The solution set is $\{2\}$.

## PROCEDURE FOR SOLVING RADICAL EQUATIONS

**Step 1:** Isolate the term with a radical on one side.

**Step 2:** Raise both (*entire*) sides of the equation to the power that will eliminate this radical, and simplify the equation.

**Step 3:** If a radical remains, repeat steps 1 and 2.

**Step 4:** Solve the resulting linear or quadratic equation.

**Step 5:** Check the solutions and eliminate any extraneous solutions.

Equations that are higher order or that have fractional powers often can be transformed into a quadratic equation by introducing a $u$-substitution. We say that equations are **quadratic in form**.

| **ORIGINAL EQUATION** | **SUBSTITUTION** | **NEW EQUATION** |
|---|---|---|
| $x^4 - 3x^2 - 4 = 0$ | $u = x^2$ | $u^2 - 3u - 4 = 0$ |
| $t^{2/3} + 2t^{1/3} + 1 = 0$ | $u = t^{1/3}$ | $u^2 + 2u + 1 = 0$ |

## PROCEDURE FOR SOLVING EQUATIONS QUADRATIC IN FORM

**Step 1:** Identify the substitution.

**Step 2:** Transform the equation into a quadratic form.

**Step 3:** Solve the quadratic equation.

**Step 4:** Apply the substitution to rewrite the solution in terms of the original variable.

**Step 5:** Solve the resulting equation.

**Step 6:** Check the solutions in the original solutions.

## Example (4)

Find the solution to the equation $x^{-2} - x^{-1} - 12 = 0$.

The solution set is $\{-\frac{1}{3}, \frac{1}{4}\}$.

## Example (5)

Find the solution to the equation $x^{2/3} - 3x^{1/3} - 10 = 0$.

The solution set is {-8, 125}.

Some equations (both polynomial and with rational exponents) that are factorable can be solved using the zero product property.

## Example (6)

Solve the equation $x^{7/3} - 3x^{4/3} - 4x^{1/3} = 0$.

The solution set is {-1, 0,4}.

## Example (7)

Solve the equation $x^3 + 2x^2 - x - 2 = 0$.

The solution set is {-2, -1, 1}.