Equations and Inequalities

1.1 Linear Equations

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Definition

A **linear equation in one variable**, x, can be written in the form

$$ax + b = 0$$

where a and b are real numbers and $a \neq 0$.

We **solve** a linear equation in one variable by finding an equivalent equation using the operations addition, subtraction, multiplication and division.

To *solve* an equation means to find all the values of x that make the equation true.

Two equations that have the same solution set are called *equivalent equations*.

Example (1)

Solve the equation $3x + 4 = 16$.

The solution set is $\{4\}$.

Example (2)

Solve the equation $5x - (7x - 4) - 2 = 5 - (3x + 2)$.

The solution set is $\{1\}$.

Example (3)

Solve the equation $\frac{1}{2}p - 5 = \frac{3}{4}p$.

The solution set is $\{-20\}$.

Example

Solve the equation

$$5x + 10 = 25 + 2x$$

Example

Solve the equation

$$5(2y - 1) = 2(4y - 3)$$

Example

Solve the equation

$$-3(4t - 5) = 5(6 - 2t)$$

Definition

A **polynomial in x** is an algebraic expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers, with $a_n \neq 0$, and n is a nonnegative integer. The polynomial is of degree n, a_n is the **leading coefficient**, and a_0 is the **constant term**.

A **rational expression** is the ratio or quotient of two polynomials.

A **rational equation** is an equation that contains one or more rational expressions.

Example (4)

Solve the equation

$$\frac{2}{3x} + \frac{1}{2} = \frac{4}{x} + \frac{4}{3}.$$

The solution set is $\{-4\}$.

Example (5)

Solve the equation

$$\frac{3x}{x-1} + 2 = \frac{3}{x-1}.$$

No Solution

Example (6)

Solve the equation

$$\frac{1}{3x+18} - \frac{1}{2x+12} = \frac{1}{x^2+6x}.$$

No Solution

Example

Dante currently has 3 test scores: 82, 79 and 90. If the score on the final exam is worth 2 test scores and his goal is to earn an 85 for his class average, what score on the final exam does dante need to achieve his course goal?

Let x equal the final exam grade. Show that $x = 87$.