September 29, 2010

Note Title 9/29/2010

You have a valid driver's license $\Rightarrow$ You have driven a vehicle

If A is True $\Rightarrow$ B is True

B True $\not\Rightarrow$ A is True

B is False $\Rightarrow$ A is False

One-to-one.

If $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

use

If $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Example

Determine whether the function is one to one

$$f(x) = |x+1|.$$

Suppose $f(x_1) = f(x_2)$

$$|x_1 + 1| = |x_2 + 1|$$

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

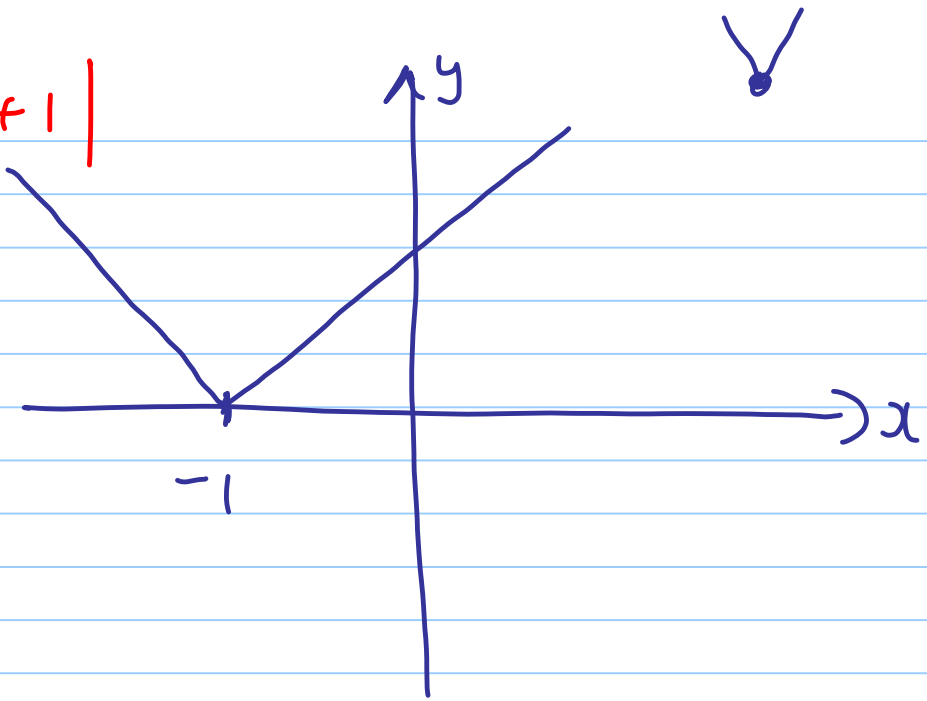
then

$$x_1 + 1 = x_2 + 1 \quad \text{OR} \quad x_1 + 1 = -(x_2 + 1)$$

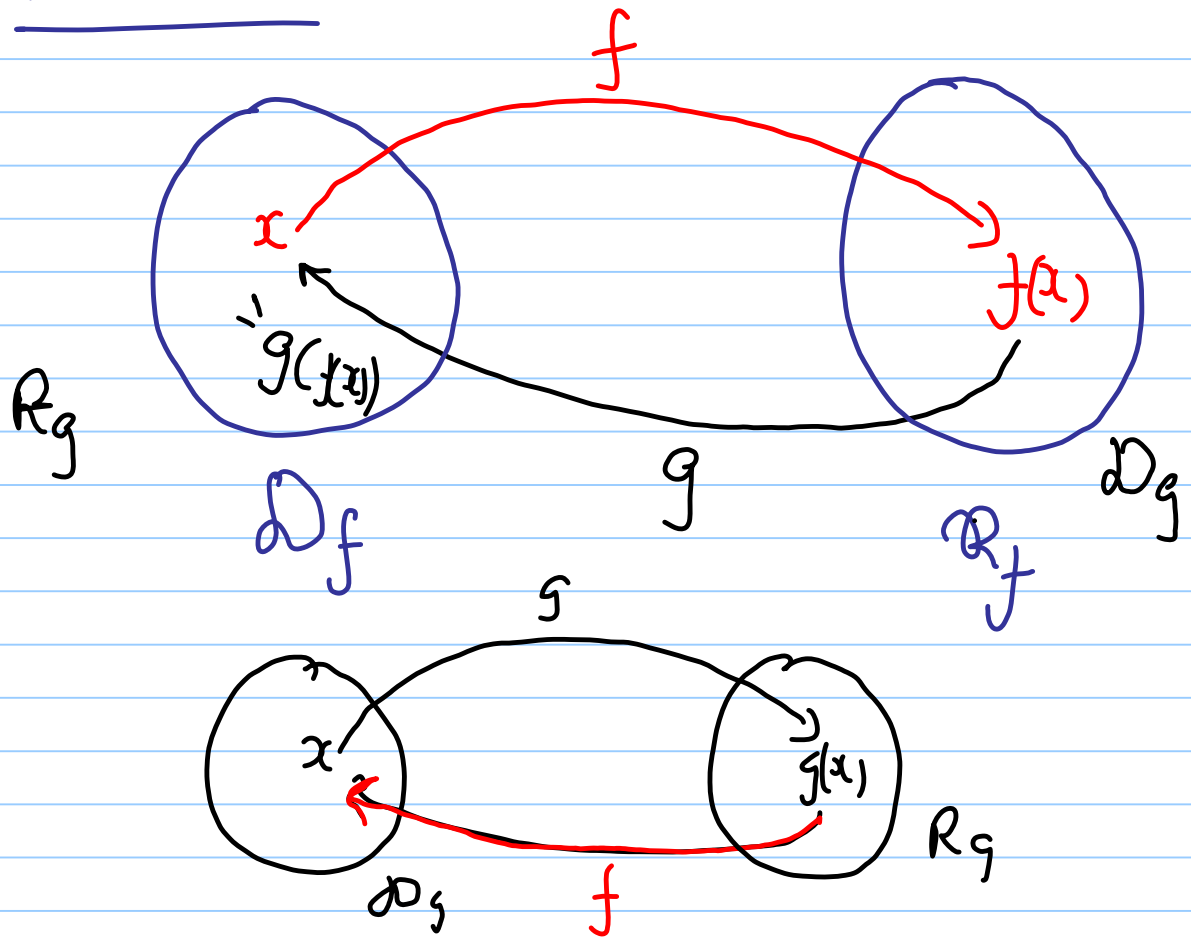
$$x_1 = x_2 \quad \text{OR} \quad x_1 = -x_2 - 2$$

NOT one-to-one

$f(x) = |x+1|$



Inverses:



Example

$f^{-1}(x) = \frac{1}{2}x - 2$

$f(x) = 2x + 4$

Show that $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

$$f(f^{-1}(x)) = f\left(\frac{1}{2}x - 2\right)$$

$$= 2\left(\frac{1}{2}x - 2\right) + 4$$

$$= x - 4 + 4$$

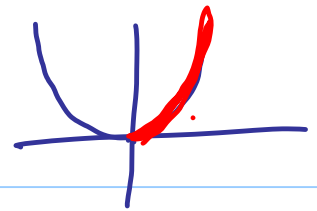
$$= x \checkmark$$

$$f^{-1}(f(x)) = f^{-1}(2x + 4)$$

$$= \frac{1}{2}(2x+4) - 2 = x + 2 - 2 = x \checkmark$$

Inverses.

Example



$f^{-1}(x) = x^2 \quad x \geq 0$

$f(x) = \sqrt{x}$

$f(f^{-1}(x)) = f(x^2)$

$= \sqrt{x^2} = x$

$f^{-1}(f(x)) = f^{-1}(\sqrt{x}) = (\sqrt{x})^2 = x$

Example

Find the inverse function of $f(x) = \sqrt{x+2}$ and state the domain and range of both f and f^{-1}.

$$f(x) = \sqrt{x+2}$$

Let $y = \sqrt{x+2}$

Interchange x and y

$$x = \sqrt{y+2}$$

Solve for y:

Square both sides

$$x^2 = y + 2$$

$$x^2 - 2 = y$$

Hence

$$f^{-1}(x) = x^2 - 2$$

Domain: $[0, \infty)$

Range: $[-2, \infty)$

$$f(x) = \sqrt{x+2}$$

Domain: $[-2, \infty)$

Range: $[0, \infty)$

Example

Find the inverse of $f(x) = |x|$ if it exists.

No inverse because it is not one to one.

Example

$f(x) = \frac{2}{x+3}$ $x \neq -3$ It is one-to-one

Let $y = \frac{2}{x+3}$

Interchange x & y

$$x = \frac{2}{y+3}$$

Solve for y.

$$x(y+3) = 2$$

$$y + 3 = \frac{2}{x}$$

$$y = \frac{2}{x} - 3$$

$$\therefore f^{-1}(x) = \frac{2}{x} - 3 \qquad x \neq 0$$

(Range $y \neq -3$ from domain of f)

$$f(x) = \frac{2}{x+3}$$

Domain $(-\infty, -3) \cup (-3, \infty)$

(Range $y \neq 0$ from domain of f^{-1})