September 27, 2010

Note Title 9/27/2010

91	92	92	92	92	92	93	93	94	95	96	96	97	98
80	81	81	81	81	82	86	86	89	89	89	89		
71	72	76	77	77	78								
61													
51	59												
40	49												

$\bar{x} = 82.11$

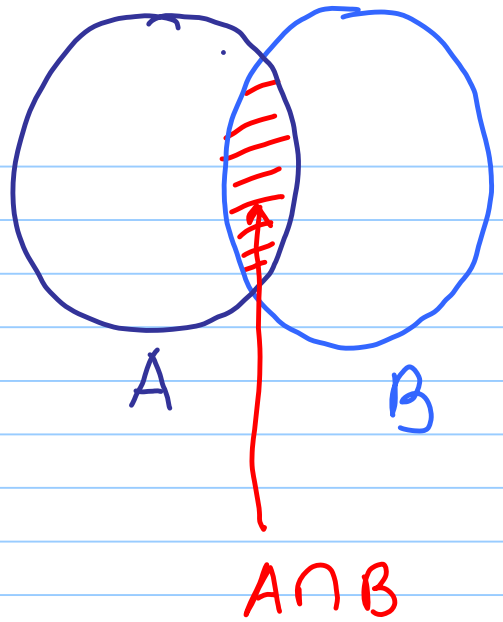
$s_x = 14.29$

$n = 37$

3.4 Operations on functions & Composition of functions.

$\cap \longrightarrow$ intersection

$A \cap B$



Example

$f(x) = \sqrt{x-1}$

$g(x) = \sqrt{4-x}$

Domain of f: $x - 1 \geq 0$

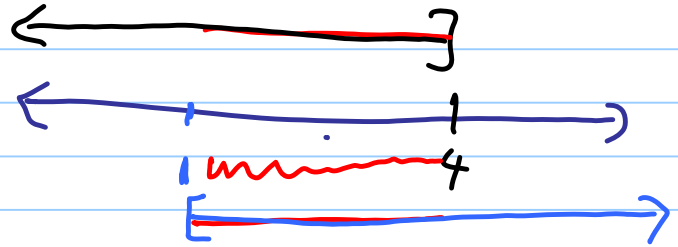
$x \geq 1$

$D_f = [1, \infty)$

Domain g: $4 - x \geq 0$

$4 \geq x$

$D_g = (-\infty, 4]$



Sum: $(f+g)(x) = f(x) + g(x)$

$= \sqrt{x-1} + \sqrt{4-x}$

Domain: $D_f \cap D_g = [1, \infty) \cap (-\infty, 4] = [1, 4]$

DIFFERENCE:

$$(f-g)(x) = f(x) - g(x)$$

$$= \sqrt{x-1} - \sqrt{4-x}$$

Domain: $D_f \cap D_g = [1,4]$

Product:

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= \left(\sqrt{x-1}\right) \cdot \sqrt{4-x}$$

$$= \sqrt{(x-1)(4-x)}$$

Domain

$[1,4]$

Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ $g(x) \neq 0$.

$$= \frac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\frac{x-1}{4-x}}$$

Domain: $D_f \cap D_g \cap \{x : g(x) \neq 0\}$

$$= [1, 4)$$

Example

$F(x) = \sqrt{x}$ $\quad D_f = [0, \infty)$

$G(x) = |x-3|$ $\quad D_G = (-\infty, \infty)$ $\quad (\mathbb{R})$

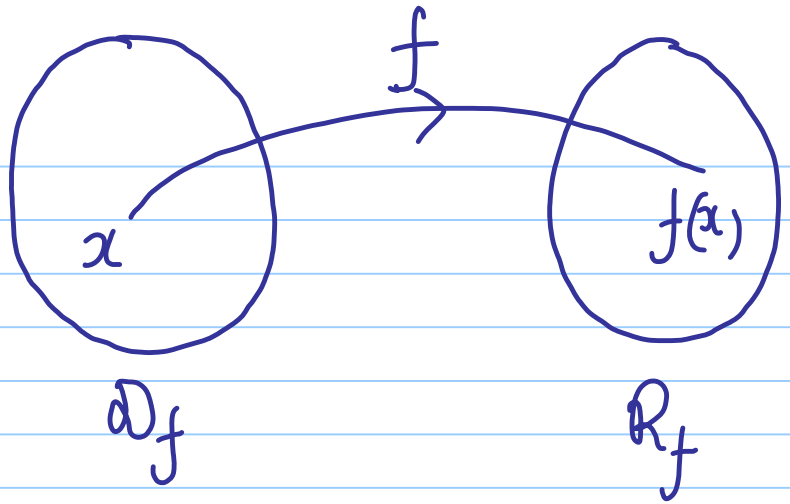
$$\left(\frac{F}{G}\right)(x) = \frac{F(x)}{G(x)}$$

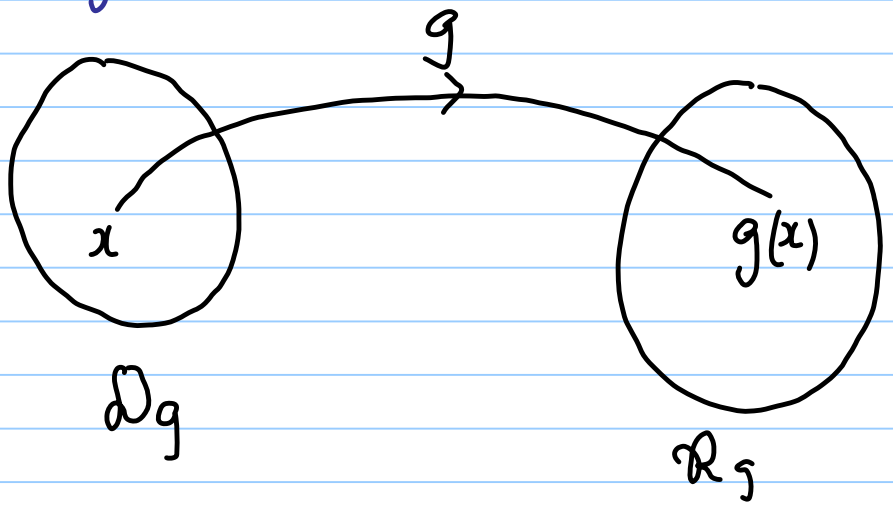
$$= \frac{\sqrt{x}}{|x-3|}$$

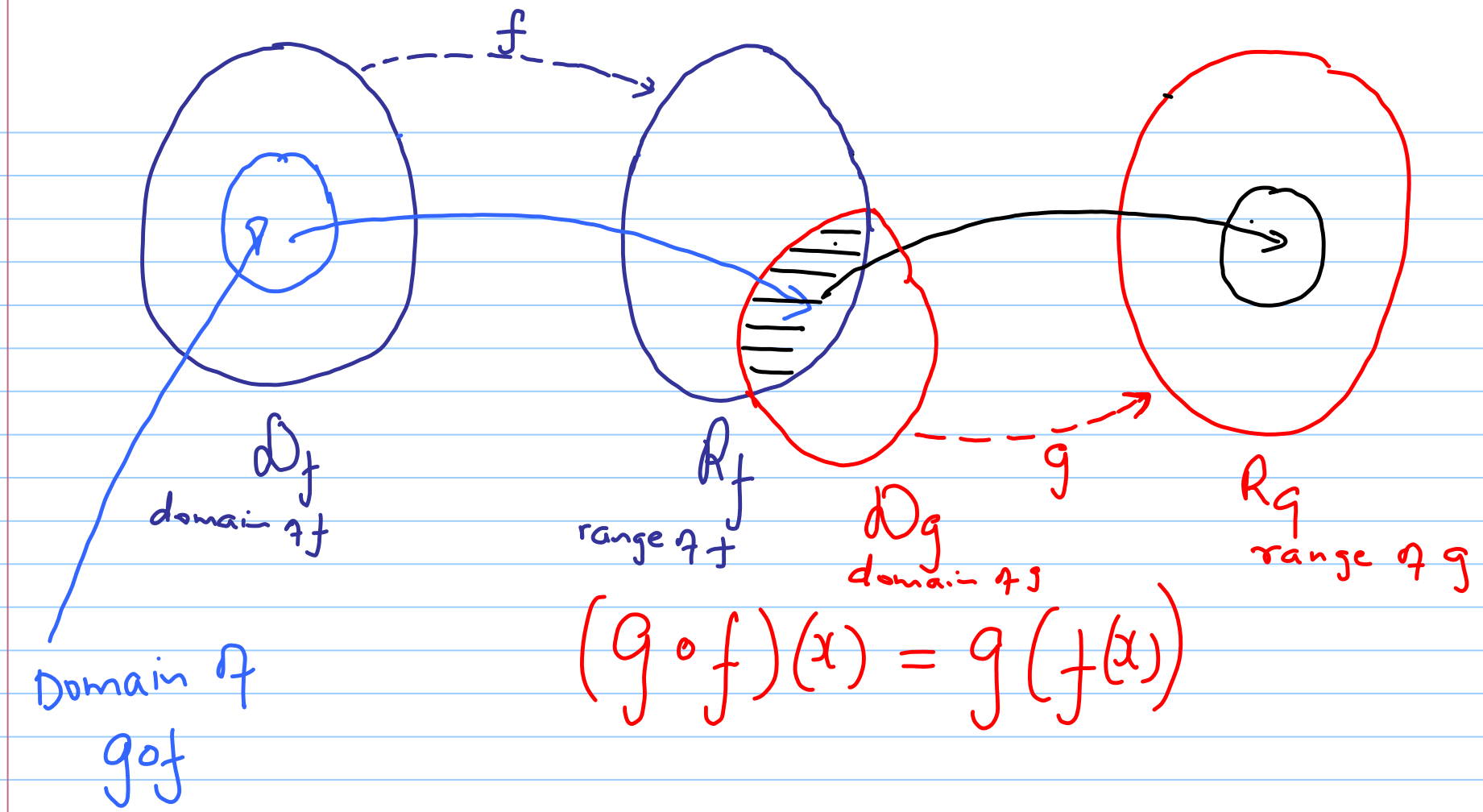
Domain: $[0, \infty) \cap (-\infty, \infty) \cap \{x : G(x) = 0\}$

$= [0, 3) \cup (3, \infty)$

Function Composition






f
D_f
domain of f
R_f
range of f
D_g
domain of g
g
R_g
range of g
Domain of g∘f
$(g \circ f)(x) = g(f(x))$

Example

$f(x) = x^2 + 1 \qquad g(x) = x - 3$

$$(f \circ g)(x) = f(g(x))$$

$$= f(x - 3)$$

$$= (x - 3)^2 + 1$$

But $(g \circ f)(x) = g(f(x))$

$= g(x^2+1)$

$= x^2+1-3$

$= x^2-2$

Observation $(f \circ g)(x) \neq (g \circ f)(x)$

Example

$f(x) = \frac{1}{x-1}$ Domain: $(-\infty, 1) \cup (1, \infty)$

$\{x \in \mathbb{R} : x \neq 1\}$

all real #s except 1.

$g(x) = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

$$(f \circ g)(x) = f(g(x))$$

$$= f\left(\frac{1}{x}\right)$$

$$= \frac{1}{\frac{1}{x} - 1}$$

$$= \frac{1}{\frac{1}{x} - \frac{x}{x}}$$

$$= \frac{1}{\frac{1-x}{x}} = \frac{x}{1-x}$$

Domain:

$g(x) \neq 1$.

$\frac{1}{x} \neq 1$

$\boxed{x \neq 1}$

all real #s except 0 & 1

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

$$(g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{1}{x-1}\right)$$

$$= \frac{1}{1/(x-1)}$$

$$= x-1$$

x	$\frac{1}{x}$
10	0.1
1	1.0
0.1	10
0.01	100
↓	
0	

Domain:

f: $x \neq 1$

g: $x \neq 0$

$f(x) \neq 0$

$\frac{1}{x-1} \neq 0$

Solve $\frac{1}{x-1} = 0$ No such x.

Domain: all real #'s

$(-\infty, 1) \cup (1, \infty)$ except 1.

Example

$f(x) = x^2 - 7 \quad g(x) = 5 - x^2$

a) $f(g(1))$

$$g(1) = 5 - 1^2 = 4$$

$$f(g(1)) = f(4) = 4^2 - 7$$
$$= 9$$

b) $f(g(-2))$ $\quad g(-2) = 5 - (-2)^2$

$= 5 - 4 = 1$

$f(g(-2)) = f(1) = 1^2 - 7$

$= -6$

c) $g(f(3))$: $\quad f(3) = 3^2 - 7 = 2$

$g(f(3)) = g(2) = 5 - 2^2 = 1$