

September 10, 2010

Note Title

9/10/2010

Equation of a straight line

Slope-intercept form: $y = mx + b$

where m is the slope

b is the y -intercept.

Example

Find the equation of line that has slope $\frac{2}{3}$ and

y -intercept $(0, 2)$.

$y = mx + b$ Find m and b !

$$y = \frac{2}{3}x + 2$$

Example

point - slope: $y - y_1 = m(x - x_1)$

point $(x_1, y_1) = (-1, 2)$

Slope $m = -\frac{1}{2}$

Equation:

$$y - 2 = -\frac{1}{2}(x - (-1))$$

↳ slope intercept form:

$$y = 2 - \frac{1}{2}(x+1)$$

$$y = 2 - \frac{1}{2}x - \frac{1}{2}$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

$$\text{Slope } m = -\frac{1}{2}$$

$$\text{Intercept } \left(0, \frac{3}{2}\right)$$

Example

Find the equation of the line through the points $(-2, -1)$ and $(3, 2)$.

Need \cdot Slope (find)

\cdot point \checkmark

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - (-1)}{3 - (-2)} = \frac{2+1}{3+2} = \frac{3}{5}$$

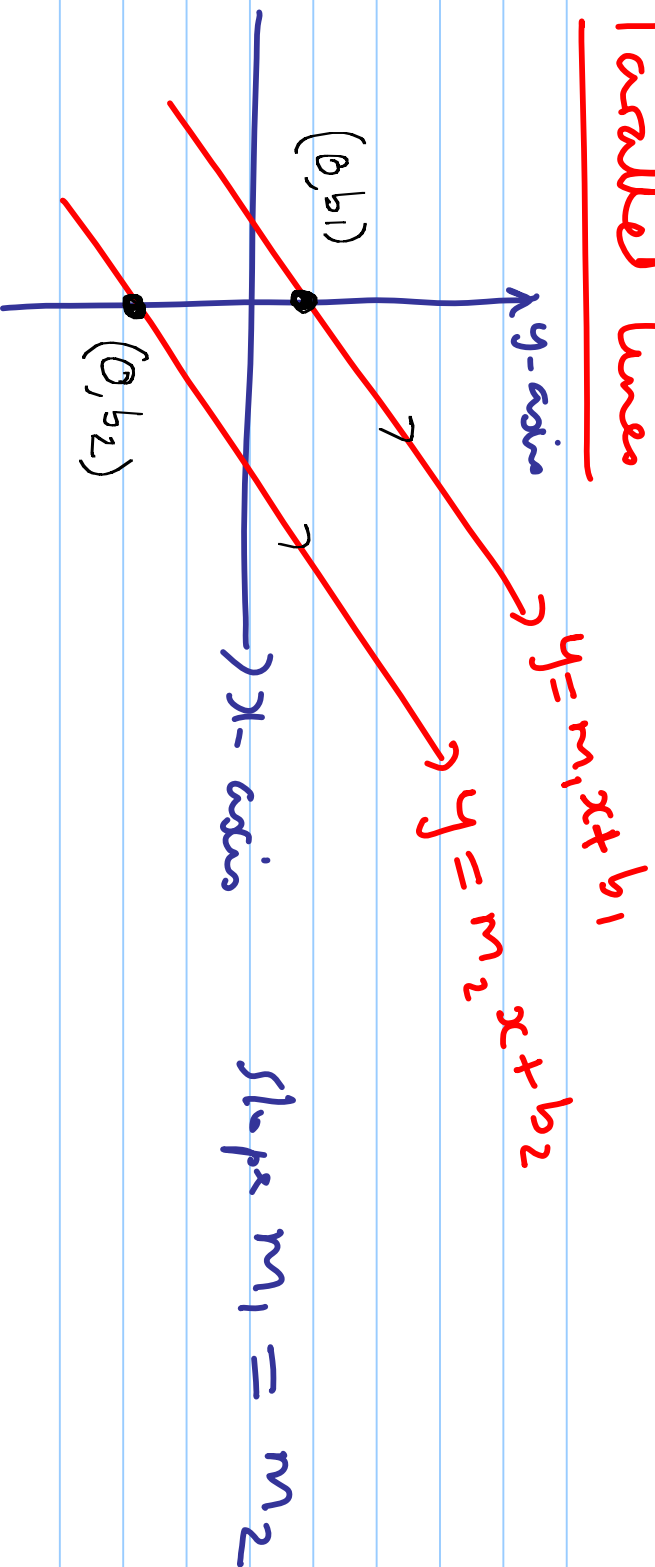
Equation of line:

$$y - y_1 = m(x - x_1) \quad \text{point-slope!}$$

$$y - (-1) = \frac{3}{5}(x - (-2))$$

$$y + 1 = \frac{3}{5}(x + 2)$$

Parallel lines



Example

Are $-x + 3y = -3$ & $y = \frac{1}{3}x - 6$ parallel?

Question. Do they have the same slope?

$$y = \frac{1}{3}x - 6 \quad \text{Slope } m = \frac{1}{3}$$

$-x + 3y = -3$ write in slope-intercept form!

$$3y = x - 3$$

$$y = \frac{1}{3}(x - 3)$$

$$y = \left[\frac{1}{3}\right]x - 1 \quad \text{Slope } m = \frac{1}{3}$$

The two lines are parallel.

Example

Given line: $y = 3x + 1$

Want equation of line through $(1, 1)$ parallel to the given line.

Know: The two lines have the same Slope.

So slope $m = 3$

Equation of line: point-slope.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 1)$$

Slope-Intercept

$$y = mx + b$$

$$y = 3x + b$$

Find b . x, y

Need point!! $(1, 1)$

$$y - 1 = 3x - 3$$

Slope-Intercept

$$y = 3x - 2$$

$$1 = 3(1) + b$$

$$1 = 3 + b$$

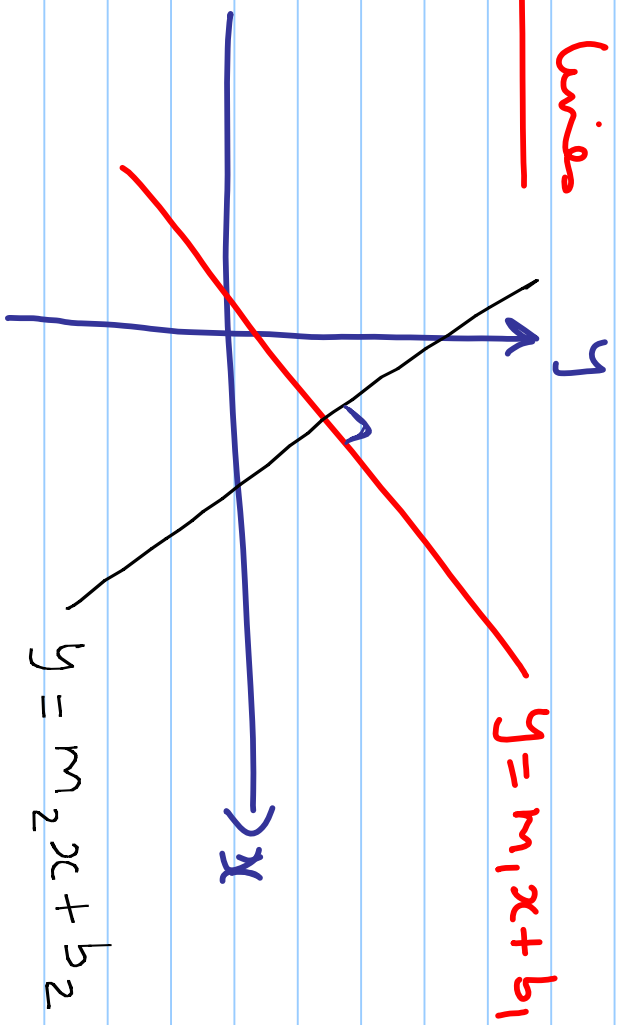
$$-2 = b$$

So

$$y = 3x - 2$$

Perpendicular lines

$$m_2 = -\frac{1}{m_1}$$



Example

Find the equation of the line that passes through the point $(3,0)$ and is perpendicular to the line

$$y = 3x + 1$$

Slope of $y = 3x + 1$ is $m = 3$.

Slope of \perp line is $m_2 = -\frac{1}{3}$

Equation of \perp line: $y - y_1 = m_2 (x - x_1)$

$$y - 0 = -\frac{1}{3} (x - 3)$$

$$y = -\frac{1}{3}(x-3)$$

Applications

$$y = mx + b$$

let C be the cost

t be the amount of time to do the job.

Service charge follows a linear equation

$$C = mt + b$$

where $m, & b$ are constant.

Have two points: $(2, 100)$ $(3, 300)$

$$\text{Slope } m = \frac{300 - 100}{3 - 2} = 200$$

$$y = 200t + b$$

↖ need

Substitute in one of the points

$$100 = 200(2) + b$$

$$100 = 400 + b$$

$$\boxed{-300 = b}$$

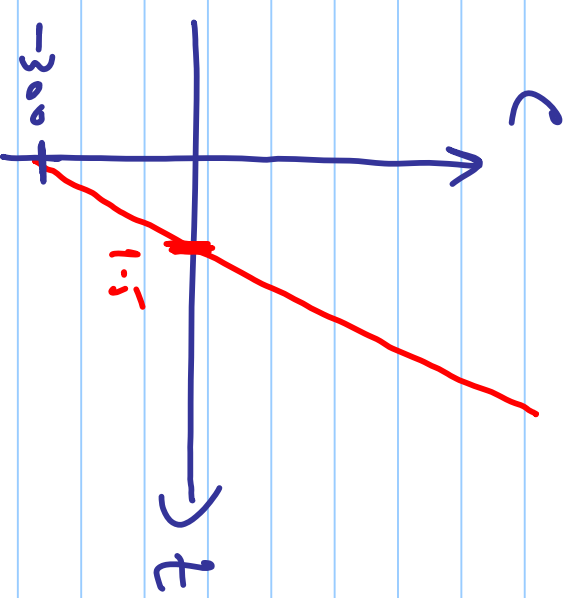
$$C = 200t - 300$$

When $t = 5$

$$C = 200(5) - 300$$

$$= 1000 - 300$$

$$= \$700$$



$$C = 200t - 300$$