October 20, 2010

Note Title 10/20/2010

§5.4 #12

$$125^x = 5^{2x-3}$$

$$(5^3)^x = 5^{2x-3}$$

$$5^{3x} = 5^{2x-3}$$

$$\Rightarrow 3x = 2x - 3$$

$$x = -3$$

Logarithmic equations.

Solve $\log_4(2x-3) = \log_4 x + \log_4(x-2)$

$$\log_4(2x-3) = \log_4\left(x(x-2)\right)$$

$\Rightarrow$ $2x-3 = x(x-2)$ one-to-one property

$$2x-3 = x^2-2x$$

$$x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=3 \quad \text{or} \quad x=1$$

Check:

$$\log_4(2(3)-3) \stackrel{?}{=} \log_4 3 + \log_4(3-2)$$

$$\log_4 3 \stackrel{?}{=} \log_4 3 + \log_4(1) \quad \checkmark$$

$x=1$:

$$\log_4(2(1)-3) \stackrel{?}{=} \log_4 1 + \log_4(1-2)$$

$$\log_4(-1) = \log_4 1 + \log_4(-1)$$

$\therefore$ Solution is $x = 3$.

Example

Solve $\log_3(9x) - \log_3(x-8) = 4$

$$\log_3\left(\frac{9x}{x-8}\right) = 4$$ Quotient rule

Change to exponential

$$\frac{9x}{x-8} = 3^4$$

$$9x = 81(x-8)$$

Divide by 9

$$x = 9(x-8)$$

$$x = 9x - 72$$

$$72 = 8x$$

$$\boxed{9 = x}$$

Example

Solve the equation $\ln(3-x^2) = 7$

Change to exponential

$$3 - x^2 = e^7$$

$$x^2 = 3 - e^7$$

$$x^2 \approx -1093$$ No real solution.

Application:

Continuous Compounding

$$P = P_0 e^{rt}$$

$P_0 = 1000$

$r = 0.05$

$t = ?$

$P = 2000$

$$2000 = 1000 e^{0.05t}$$

$$2 = e^{0.05t}$$

$$\ln 2 = 0.05t$$

$$t = \frac{\ln 2}{0.05} \approx 14 \text{ years!!}$$