October 19, 2010

Note Title 10/19/2010

§5.3 #40

$$3\log u - \log 2v - \log z = \log u^3 - (\log 2v + \log z)$$

$$= \log u^3 - \log(2v \cdot z)$$

$$= \log\left(\frac{u^3}{2vz}\right)$$

5.4 Exponential Equations

Example Solve

a) $3^x = 81$

$3^x = 3^4$

$b^x = b^y$ iff $x = y$

by one to one property

$\boxed{x = 4}$

b) $5^{7-x} = 125$

$5^{7-x} = 5^3$

$\Rightarrow 7 - x = 3$

$7 - 3 = x$

$\boxed{4 = x}$

Check: $5^{7-4} = 5^3 = 125$

c) $\left(\frac{1}{2}\right)^{4x} = 16$

$\left(2^{-1}\right)^{4x} = 16$

$2^{-4x} = 16$

$2^{-4x} = 2^{4}$

$\Rightarrow -4x = 4$

$\boxed{x = -1}$

Example Solve the exponential equation and round answer to 4 decimal places.

(a) $5^{3x} = 16$

Take natural logarithms of both sides

$$\ln 5^{3x} = \ln 16$$

Power rule

$$3x \cdot (\ln 5) = \ln 16$$

$$x = \frac{\ln 16}{3 \ln 5} \approx 0.5742$$

b) $4^{3x+2} = 71$

Take log of both sides

$$\log 4^{3x+2} = \log 71$$

$$(3x+2)\log 4 = \log 71$$

$$3x+2 = \frac{\log 71}{\log 4}$$

$$3x = \frac{\log 71}{\log 4} - 2$$

$$x = \left(\frac{\log 71}{\log 4} - 2\right)\frac{1}{3}$$

$$\approx 0.3583$$

Example

Solve the exponential equation

$$4e^{x^2} = 64$$

Divide both sides by 4

$$e^{x^2} = 16$$

Take natural logs

$$\ln e^{x^2} = \ln 16$$

$$x^2 \ln e = \ln 16$$

$$x^2 (1) = \ln 16$$

$$x = \pm\sqrt{\ln 16}$$

$$\approx \pm 1.6651 \quad \text{(four decimal places)}$$

Example

Solve the exponential equation

$$e^{2x} - 4e^{x} + 3 = 0 \qquad (b^m)^n = b^{mn}$$

Resembles quadratic

$$(e^x)^2 - 4(e^x) + 3 = 0$$

Let $u = e^x$

$u^2 - 4u + 3 = 0$ Quadratic

$(u-3)(u-1) = 0$

either $u = 3$ or $u = 1$

So $e^x = 3$

$x = \ln 3$

≈ 1.0986

$e^x = 1$

$x = 0$