Note Title 10/18/2010

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#32) $e^{-x} = 4$ $y = \log_b x \iff x = b^y$

the base e is a number.

$$-x = \log_e 4$$

$$-x = \ln 4$$

logarithm properties:

<u>Examples</u>

a) $\log_{10} 10 = 1$ prop 2 $\log_b b = 1$

b) $\ln 1 = 0$ prop 1 $\log_b 1 = 0$

c) $10^{\log\boxed{(x+8)}} = x+8$ prop 4 $b^{\log_b \boxed{x}} = x$

d) $e^{\ln(2x+5)} = 2x+5$

e) $\log 10^{x^2} = x^2$

f) $\ln e^{x+3} = x+3$

<u>Example</u>

Write expression $\log_b (u^2 \sqrt{v})$ as sum of simpler logarithms.

$$\log_b (u^2 \sqrt{v}) = \log_b u^2 + \log_b \sqrt{v}$$ product rule

$$= \log_b u^2 + \log_b v^{1/2}$$

$$= 2\log_b u + \frac{1}{2}\log_b v \quad \text{power rule}$$

<u>Example</u>

write $\ln\left(\frac{x^3}{y^2}\right)$ as a difference of logs.

$$\ln\left(\frac{x^3}{y^2}\right) = \ln x^3 - \ln y^2 \quad \text{Quotient rule}$$

$$= 3\ln x - 2\ln y \quad \text{power rule.}$$

Example

$\ln\left(\frac{x^2-x-6}{x^2+7x+6}\right)$ write as a sum or difference of logs

$$\ln\left(\frac{x^2-x-6}{x^2+7x+6}\right) = \ln(x^2-x-6) - \ln(x^2+7x+6)$$

$$= \ln((x+2)(x-3)) - \ln((x+6)(x+1))$$

$$= \ln(x+2) + \ln(x-3) - [\ln(x+6) + \ln(x+1)]$$

$$= \ln(x+2) + \ln(x-3) - \ln(x+6) - \ln(x+1)$$

<u>Example</u> Rewrite as a single logarithm expression

$$\frac{2}{3}\ln x - \frac{1}{2}\ln y = \ln x^{2/3} - \ln y^{1/2}$$ power rule

$$= \ln\left(\frac{x^{2/3}}{y^{1/2}}\right)$$ Quotient rule

Example

$$\frac{1}{2}\log_b x + \log_b(2x+1) - 2\log_b 4$$

$$= \log_b x^{1/2} + \log_b(2x+1) - \log_b 4^2 \quad \text{power rule}$$

$$= \log_b x^{1/2}(2x+1) - \log_b 16 \quad \text{product rule}$$

$$= \log_b\left(\frac{\sqrt{x}\,(2x+1)}{16}\right) \quad \text{quotient rule}$$

Change of base.

$$\log_4 17 = \frac{\log_{10} 17}{\log_{10} 4}$$

log(17)/log(4)

$$\approx 2.0437$$

$$\log_4 17 = \frac{\ln(17)}{\ln 4} \approx 2.0437$$

Compound interest

$$P = P_0\left(1 + \frac{r}{n}\right)^{nt}$$

Monthly $P = P_0\left(1 + \frac{r}{12}\right)^{12t}$

Quarterly $P = P\left(1 + \frac{r}{4}\right)^{4t}$

Daily $P = P\left(1 + \frac{r}{365}\right)^{365t}$

When $n \longrightarrow \infty$ very large

Continuous compounding

$$P = P_0 e^{rt} \qquad \left(1 + \frac{r}{n}\right)^n \longrightarrow e$$

$$n \to \infty$$