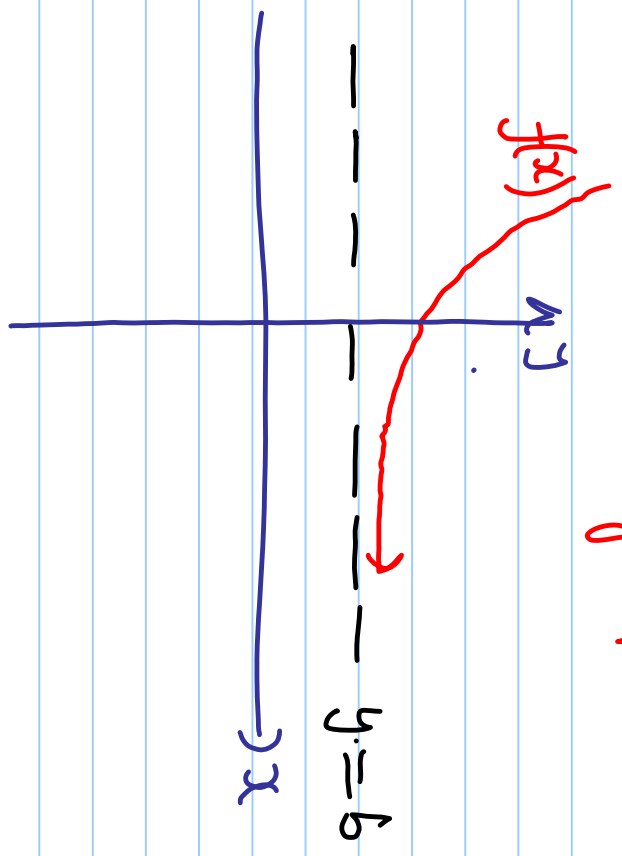
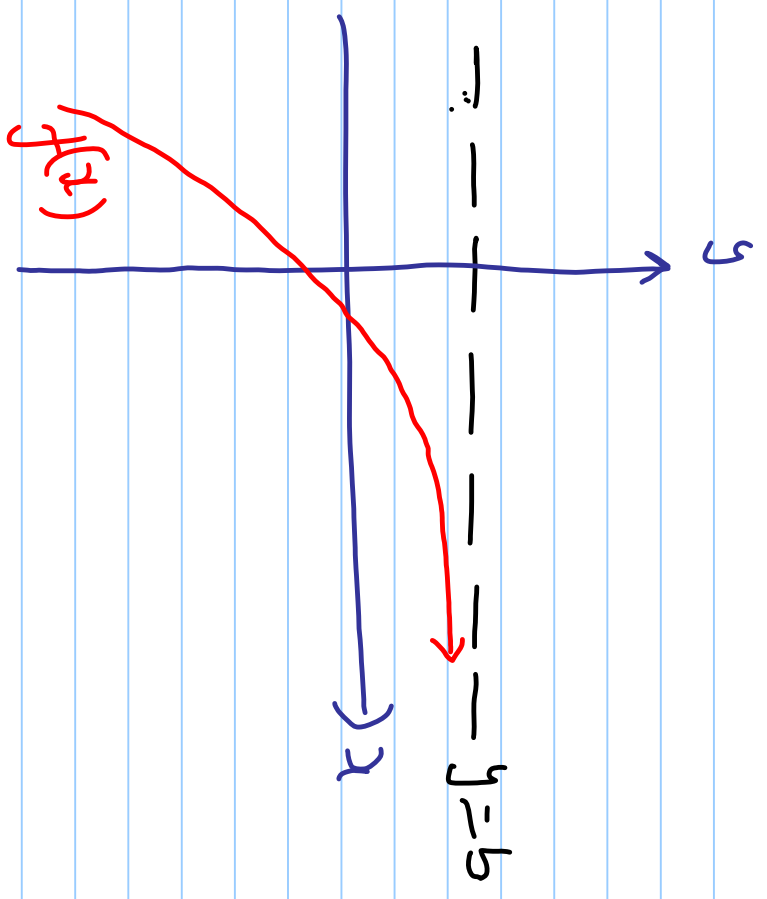




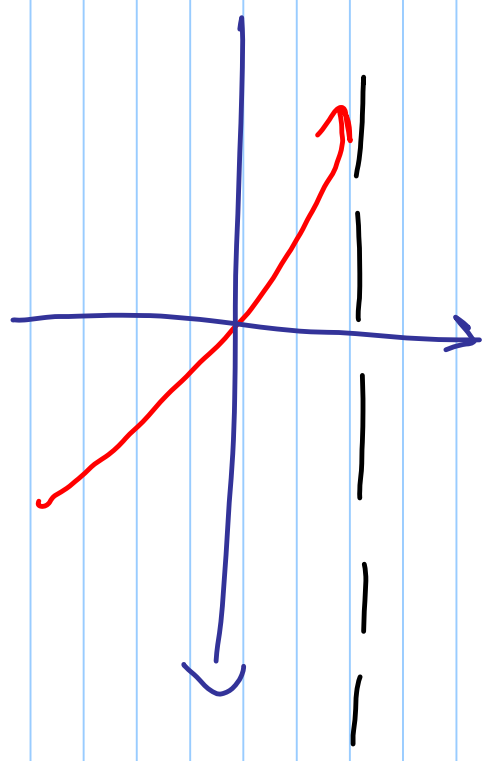
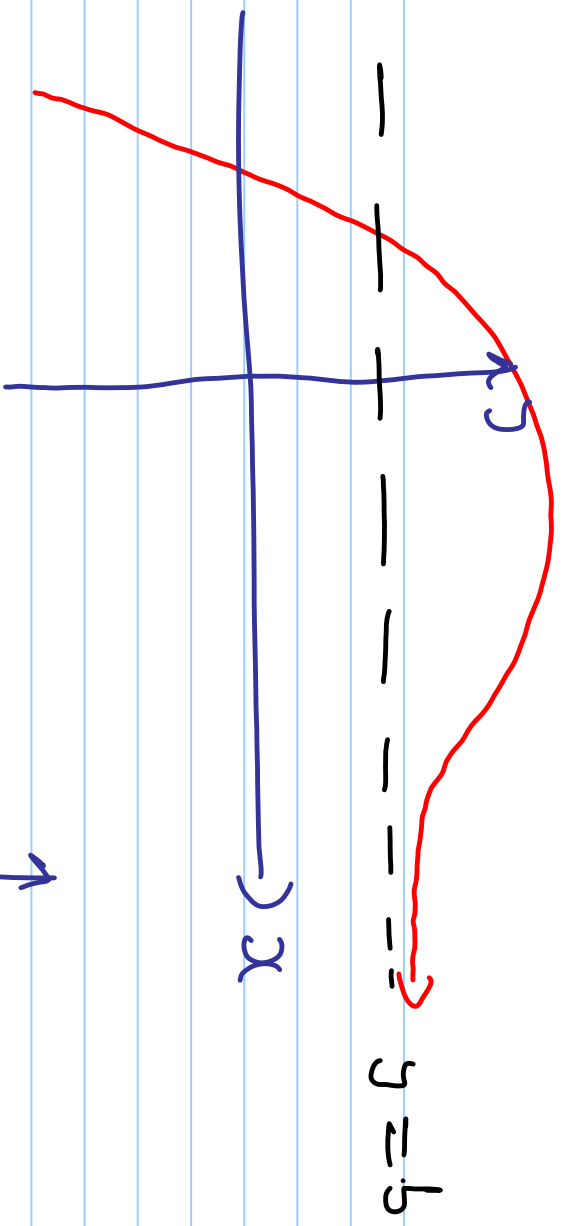
# Horizontal Asymptotes . . .



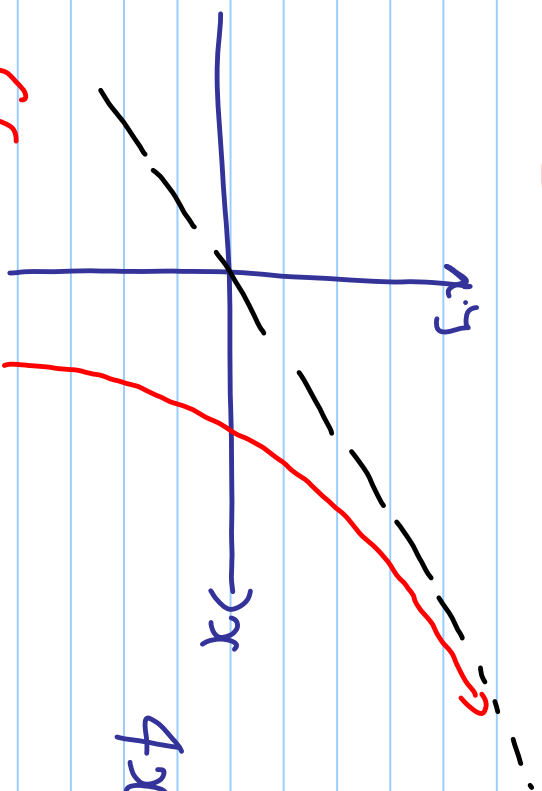
As  $x \rightarrow \infty$



$f(x) \rightarrow b$



# SLANT ASYMPTOTE



Aside

$$\begin{array}{r} 66 \\ 3 \overline{) 199} \\ \underline{18} \phantom{0} \\ 19 \phantom{0} \\ \underline{18} \phantom{0} \\ 1 \phantom{0} \end{array}$$

$\frac{199}{3} = 66 + \frac{1}{3}$  ← remainder

$$h(x) = \frac{8x^3 + 3}{4x^2 + 1}$$

$$\begin{array}{r} 2x \\ \hline 8x^3 + 0x^2 + 0x + 3 \\ \underline{- 2x} \phantom{+ 3} \\ -2x + 3 \end{array}$$

Remainder

$$\frac{8}{5} = 1 + \frac{3}{5}$$

$$\frac{8x^3+3}{4x^2+1} = \boxed{2x} + \frac{-2x+3}{4x^2+1}$$

Slant  
Asymptote

Example

$$f(x) = \frac{x^2+9x+20}{x-3}$$

$$x-3 \overline{) x^2 + 9x + 20}$$

$$- x^2 - 3x \quad \downarrow$$

$$\underline{12x + 20}$$

$$\underline{12x - 36}$$

56R  
Remainder

∴ Slant asymptote

$$y = x + 12.$$

$$\frac{x^2 + 9x + 20}{x-3} = \boxed{x+12} + \frac{56}{x-3}$$

## Exponential functions

$$f(2) = 3^2 = 9$$

$$x^{-n} = \frac{1}{x^n}$$

$$g\left(-\frac{3}{2}\right) = \left(\frac{1}{4}\right)^{-\frac{3}{2}}$$

$$= \left(\frac{4}{1}\right)^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = (2)^3 = 8$$