7.7 Inverse Trigonometric Functions

Note Title 11/30/2010

$\sin(\sin^{-1} x) = x \qquad -1 \le x \le 1.$

$\sin^{-1}(\sin x) = x \qquad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Example

$\sin^{-1}\left[\sin\frac{3\pi}{4}\right]$ Can not use the inverse identities.

Note that

$$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

Then

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

Hence $\sin^{-1}\left[\sin\frac{3\pi}{4}\right] = \frac{\pi}{4}$

If you want to apply the inverse identity.

Note that. $\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$

$$\therefore \sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$$

Example

$$\cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$$

not in the interval $[0, \pi]$

Example

$$\tan\left(\tan^{-1} 17\right) = 17$$

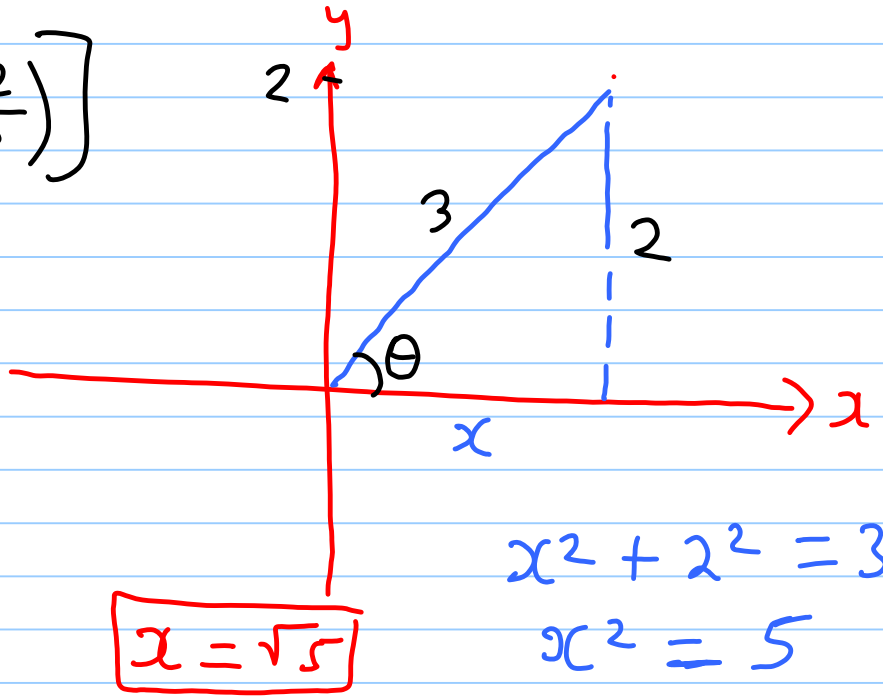
$$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$$

Example

Find the exact value of

$$\cos\left[\sin^{-1}\left(\frac{2}{3}\right)\right]$$

Let $\theta = \sin^{-1}\left(\frac{2}{3}\right)$



$x^2 + 2^2 = 3^2$

$x^2 = 5$

$x = \sqrt{5}$

$\cos\theta = \frac{x}{r}$

$= \frac{\sqrt{5}}{3}$

7.8 Trigonometric Equations

Example Solve over $[0; 2\pi]$

$$\cos x = \frac{1}{2}$$

Cosine is positive in QI & QIV

So $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

Example

$$\sin(2x) = \frac{1}{2}$$

Sine is positive in QI & QII

So $2x = \frac{\pi}{6}$ or $2x = \frac{5\pi}{6}$

$$\boxed{x = \frac{\pi}{12}} \qquad \boxed{x = \frac{5\pi}{12}}$$

Example

Solve $\cos x = \frac{\sqrt{2}}{2}$.

Cosine is positive in QI & QIV

$x = \frac{\pi}{4}$ or $x = \frac{7\pi}{4}$

but cosine function has period 2π

So

$$x = \frac{\pi}{4} + 2\pi n \quad \text{or} \quad x = \frac{7\pi}{4} + 2\pi n$$

$$= \frac{\pi(1 + 8n)}{4} \qquad = \frac{(7 + 8n)\pi}{4}$$

n is an integer

Example

Solve $\tan 2x = -\frac{\sqrt{3}}{3}$

Tangent is negative in QII & QIV

period for tangent function is π

$\therefore 2x = \frac{5\pi}{6} + \pi n$ n is an integer

$x = \frac{1}{2}\left(\frac{5\pi}{6} + \pi n\right)$