Half Angle Identities

Note Title 11/18/2010

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} \quad \text{or} \quad \frac{\sin A}{1+\cos A} \quad \text{or} \quad \frac{1-\cos A}{\sin A}$$

Example

Verify the identities

a) $\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \cos x$

$$\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2} - \frac{1-\cos x}{2}$$

$$= \frac{1+\cos x - 1 + \cos x}{2}$$

$$= \frac{2\cos x}{2} = \cos x$$

b) $\tan\left(\frac{A}{2}\right) + \cot\left(\frac{A}{2}\right) = 2\csc A$

$\cot\left(\frac{A}{2}\right) = \frac{1}{\tan\left(\frac{A}{2}\right)}$

$$\tan\left(\frac{A}{2}\right) + \cot\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A} + \frac{\sin A}{1-\cos A}$$

$$= \sin A(1-\cos A)$$

$$= \frac{(1-\cos A)^2 + \sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{1 - 2\cos A + \cos^2 A + \sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{1 - 2\cos A + 1}{\sin A(1-\cos A)}$$

$$= \frac{2 - 2\cos A}{\sin A(1-\cos A)}$$

$$= \frac{2(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{2}{\sin A} = 2\csc A$$

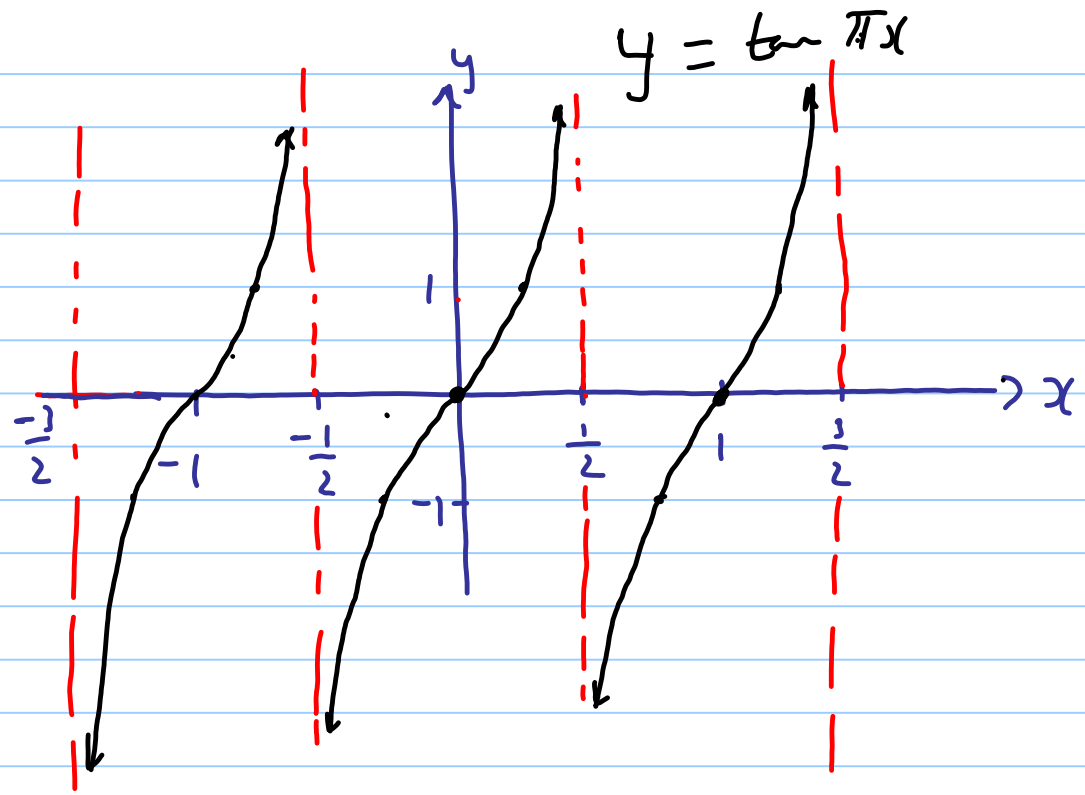
Example

$$y = \frac{\sin 2\pi x}{1+\cos 2\pi x}$$

Consider $y = \tan \pi x = \frac{\sin 2\pi x}{1+\cos 2\pi x}$

vertical asymptotes:

x	$y = \tan \pi x$	Point
$-1/2$	undefined	
$-1/4$	-1	$(-1/4, -1)$
0	0	$(0, 0)$
$\frac{1}{4}$	1	$(\frac{1}{4}, 1)$
$\frac{1}{2}$	undefined	VA
$\frac{3}{4}$	-1	$(\frac{3}{4}, -1)$
1	0	$(1, 0)$



$\cos A = 0$ $\quad A = \frac{\pi}{2}$ $\quad \cos \pi x = 0$ $\quad \pi x = \frac{\pi}{2}$