Questions § 7.2

Note Title 11/15/2010

#30] $\csc x - \sin x = \cot x \cos x$

$$\csc x - \sin x = \frac{1}{\sin x} - \sin x$$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x} = \frac{\cos x}{\sin x} \cdot \cos x = \cot x \cos x$$

#42] $\dfrac{\cos^2 x}{1-\sin x} = 1+\sin x$

$$\frac{\cos^2 x}{1-\sin x} = \frac{1-\sin^2 x}{1-\sin x}$$

$$= \frac{(1+\sin x)(1-\sin x)}{1-\sin x}$$

$$= 1+\sin x .$$

§7.3 Sum and Difference Identities

Example

1 $\cos\frac{\pi}{12} = ?$ Write $\frac{\pi}{12}$ as a sum or difference of "nice" angles

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4} \qquad \frac{4\pi}{4\cdot 3} - \frac{3\pi}{3\cdot 4}$$

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}.$$

(2) $\cos(195^\circ) = \cos(150^\circ + 45^\circ)$

$= \cos(150^\circ)\cos(45^\circ) - \sin(150^\circ)\sin(45^\circ)$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= -\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$

Example

$$\cos(4x)\cos(7x) + \sin(4x)\sin 7x = \cos(4x - 7x)$$

$$= \cos(-3x)$$

$$= \cos(3x)$$

Cofunction Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \underbrace{\cos\frac{\pi}{2}}_{0}\cos\theta + \underbrace{\sin\frac{\pi}{2}}_{1}\sin\theta$$

$$= \sin\theta$$

Example

$$\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$

$$= \left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}\right)+\left(\cos\frac{\pi}{3}\right)\left(\sin\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}$$