# 7.2: Verifying Trigonometric Identities:

Note Title 11/11/2010

We will use:

- Reciprocal identities
- Quotient identities
- Pythagorean identities
- Even-odd identities

- $\begin{cases} \sin(-x) = -\sin x \\ \csc(-x) = -\csc x \\ \tan(-x) = -\tan x \\ \cot(-x) = -\cot x \end{cases}$ $\begin{cases} \cos(-x) = \cos x \\ \sec(-x) = \sec x \end{cases}$

# Simplifying Trigonometric Expressions using Identities

## Example

Simplify $\tan x \sin x + \cos x$

$$\tan x \sin x + \cos x = \frac{\sin x}{\cos x} \sin x + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \cos x$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x} = \sec x$$

## Example

Simplify $\cot x \cos x + \sin x$

$$\cot x \cos x + \sin x = \frac{\cos x}{\sin x} \cdot \cos x + \sin x$$

$$= \frac{\cos^2 x}{\sin x} + \sin x$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} = \csc x.$$

## Example

Simplify $\frac{1}{\csc^2 x} + \frac{1}{\sec^2 x}$

$$\frac{1}{\csc^2 x} + \frac{1}{\sec^2 x} = \left(\frac{1}{\csc x}\right)^2 + \left(\frac{1}{\sec x}\right)^2$$

$$= (\sin x)^2 + (\cos x)^2$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

# Example

Simplify $\frac{1}{\cos^2 x} - 1$

$$\frac{1}{\cos^2 x} - 1 = \sec^2 x - 1$$

$$= \tan^2 x$$

$\tan^2 x + 1 = \sec^2 x$

OR $\frac{1}{\cos^2 x} - 1 = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

$$= \frac{1 - \cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} = \left(\frac{\sin x}{\cos x}\right)^2$$

$$= \tan^2 x$$

## Example

$(A-B)(A+B) = A^2 - B^2$

Simplify the trigonometric expressions

a) $(\sin x - \cos x)(\sin x + \cos x)$

$= \sin^2 x - \cos^2 x$ $\quad \sin^2 x + \cos^2 x = 1 \therefore$

$= \sin^2 x - (1 - \sin^2 x)$ or $(1 - \cos^2 x) - \cos^2 x$

$= \sin^2 x - 1 + \sin^2 x$ $\quad 1 - 2\cos^2 x$

$= 2\sin^2 x - 1$

b) $\dfrac{1-\cot(-x)}{1+\cot x} = \dfrac{1-(-\cot x)}{1+\cot}$

$= \dfrac{1+\cot x}{1+\cot x} = 1$

c) $\dfrac{\sec x}{\tan x} = \dfrac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \dfrac{1}{\cos x} \cdot \dfrac{\cos x}{\sin x}$

$= \dfrac{1}{\sin x} = \csc x$

d) $\dfrac{1-\cot^4(x)}{1-\cot^2 x} \overset{\text{Factor}}{=} \dfrac{(1-\cot^2 x)(1+\cot^2 x)}{1-\cot^2 x}$

$= 1+\cot^2 x$

$= \csc^2 x$

e) $1-\dfrac{\cos^2 x}{1+\sin x} = \dfrac{1+\sin x-\cos^2 x}{1+\sin x}$

$$= \frac{(1 - \cos^2 x) + \sin x}{1 + \sin x}$$

$$= \frac{\sin^2 x + \sin x}{1 + \sin x}$$

$\sin^2 x + \cos^2 x = 1$

$$= \frac{\sin x (\sin x + 1)}{1 + \sin x}$$

Factor

$$= \sin x$$