Note Title 11/10/2010

100 100 100

92 93 93.5 95 96 96 96 97 98 98 98

80 84 86 # 86 88

70 71 72 72 73 73.5 76 76

65 66 68

50.5 54

$\bar{x} = 83.23$

$s_x = 14.26$

$n = 32$

7.1 Reciprocal Identities

Example

a) If $\sin\theta = -\frac{3}{7}$, find $\csc\theta$.

$$\csc\theta = \frac{1}{\sin\theta}$$

$$= \frac{1}{-3/7} = -\frac{7}{3}$$

b) If $\cos\theta = 0.8$, find $\sec\theta$.

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{0.8} = \frac{1}{8/10} = \frac{10}{8} = \frac{5}{4} \quad (\text{or } 1.25)$$

c) If $\tan\theta = 0.5$, find $\cot\theta$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \frac{1}{0.5} = 2$$

d) If $\sec\theta = \frac{\sqrt{11}}{2}$, find $\cos\theta$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{\sqrt{11}/2} = \frac{2}{\sqrt{11}} = \frac{2\sqrt{11}}{11}$$

e) If $\cot\theta = 3.5$, find $\tan\theta$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$= \frac{1}{3.5} = \frac{1}{7/2} = \frac{2}{7}$$

Example

a) If $\sin\theta = -\frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$, find $\cot\theta$.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

$$\frac{\sqrt{3}/2}{-1/2} = -\frac{\sqrt{3}}{2} \div \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$$

b) If $\sin\theta = -0.6$ and $\cos\theta = -0.8$

find $\tan\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-0.6}{-0.8} = \frac{6}{8} = \frac{3}{4} \text{ (or } 0.75\text{)}$$

c) If $\sin\theta = -\frac{\sqrt{11}}{6}$ and $\cos\theta = -\frac{5}{6}$, find $\cot\theta$.

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{-5/6}{-\sqrt{11}/6} = \frac{5}{6} \div \frac{\sqrt{11}}{6} = \frac{5}{6} \cdot \frac{6}{\sqrt{11}} = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

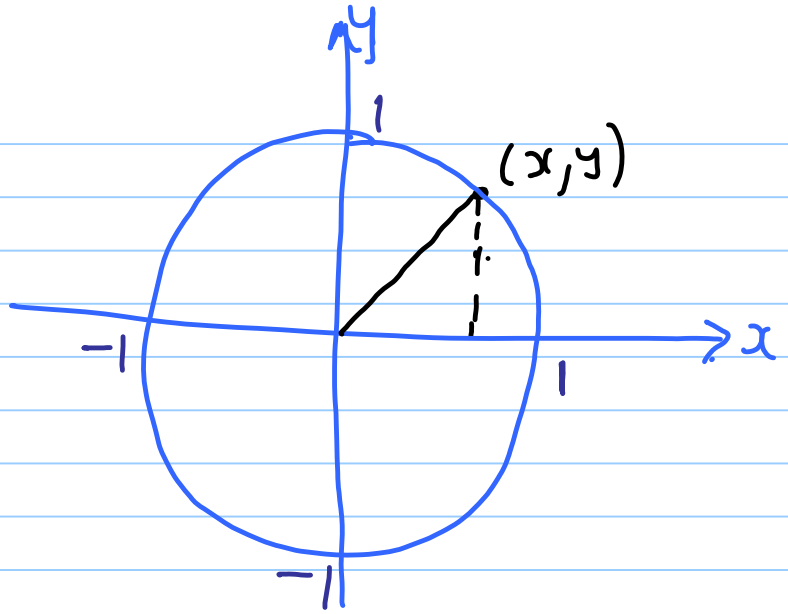
Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \quad (*)$$

If we divide $(*)$ by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1 = \left(\frac{1}{\cos\theta}\right)^2$$



$$x^2 + y^2 = r^2 = 1$$

$$x = r\cos\theta = \cos\theta$$

$$y = \sin\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{1}{\cos^2\theta} = \frac{1}{(\cos\theta)^2}$$

Similarly

$$1 + \cot^2\theta = \csc^2\theta$$

Example

a) If $\sin\theta = -\frac{3}{5}$ and the terminal side of θ lies in Q III, find $\cos\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(-\frac{3}{5}\right)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{9}{25}$$

$$\cos^2\theta = \frac{16}{25}$$

$$\cos\theta = \pm\sqrt{\frac{16}{25}}$$

$$= \pm\frac{4}{5}$$

In Q III Cosine function is negative $\therefore \boxed{\cos\theta = -\frac{4}{5}}$

Example

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = -\frac{4}{3}$ and the terminal side of θ lies in QII.

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(-\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\frac{25}{9} = \sec^2\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{-4}{3}$$

$$\Rightarrow \frac{25}{9} = \frac{1}{\cos^2\theta}$$

$$\cos^2\theta = \frac{9}{25}$$

$$\cos\theta = \pm\sqrt{\frac{9}{25}}$$

$$= \pm\frac{3}{5}$$

In QII $\quad \cos\theta = -\frac{3}{5}$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(-\frac{3}{5}\right)^2 + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$= \frac{16}{25}$$

$$\sin\theta = \pm\sqrt{\frac{16}{25}}$$

$$= \pm\frac{4}{5}$$

In QII, $\sin\theta = \frac{4}{5}$

Example

Perform the indicated operations and simplify your answers, if possible. (answers in terms $\sin\theta$ & $\cos\theta$).

a) $\sec\theta \cot\theta = \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

b) $\tan^2\theta - \sec^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$$= \frac{\sin^2\theta - 1}{\cos^2\theta}$$

$$= -\frac{\cos^2\theta}{\cos^2\theta}$$

$$= -1$$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta - 1 = -\cos^2\theta$