Questions

Note Title 12/3/2010

§7.8 #91

$\sin\left(\frac{\theta}{2}\right) = -\frac{1}{2}$ Interval: all real numbers.

Sine is negative in QIII & QIV

In QIII

$\frac{\theta}{2} = \frac{7\pi}{6}$

In QIV:

$\frac{\theta}{2} = \frac{11\pi}{6}$

all real numbers:

$\frac{\theta}{2} = \frac{7\pi}{6} + 2\pi n$

$\theta =$

$\frac{\theta}{2} = \frac{11\pi}{6} + 2\pi n$

$\theta =$

Half angle identity

$\theta = \pi/4$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

a) $$\sin\left(\frac{\pi}{8}\right) = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

§7.8 #29 $\tan^2\theta - 1 = 0$ $0 \le \theta < 2\pi$

$\tan^2\theta - 1 = 0$

$(\tan\theta - 1)(\tan\theta + 1) = 0$

either $\tan\theta - 1 = 0$ or $\tan\theta + 1 = 0$

$\tan\theta = 1$ $\tan\theta = -1$

$\theta = \frac{\pi}{4}$ or $\theta = \frac{5\pi}{4}$ $\theta = \frac{3\pi}{4}$ or $\theta = \frac{7\pi}{4}$

§8.2 #11

Solve the triangle $b = 7, c = 2, \alpha = 16^\circ$

Find a

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$a^2 = 7^2 + 2^2 - 2(7)(2)\cos 16^\circ$$

$$a^2 = 49 + 4 - 28\cos 16^\circ$$

$$a^2 = 26.0847$$

$$a = \pm\sqrt{26.0847} \simeq \pm 5.107$$

take $\boxed{a \approx +5}$ one significant digit

Find β:

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$$

$$\frac{\sin\beta}{7} = \frac{\sin 16^\circ}{5}$$

$$\sin\beta = \frac{7\sin 16^\circ}{5}$$

$$\beta = \sin^{-1}\left(\frac{7\sin 16^\circ}{5}\right) \approx 22.69$$

$\beta \simeq 23^\circ$ two significant digits

Hence $\gamma = 180^\circ - (\alpha + \beta)$

$= 180^\circ - (23^\circ + 16^\circ)$

$\gamma = 141^\circ$

§7.8 #25

$2\cos(2\theta) + 1 = 0 \qquad 0 \le \theta < 2\pi$

$2\cos(2\theta) = -1$

$\cos(2\theta) = -\frac{1}{2}$

$2\theta = \frac{4\pi}{3}$ or $2\theta = \frac{2\pi}{3}$

$\theta = \frac{4\pi}{6}$

$= \frac{2\pi}{3}$ $n = 0$

$\boxed{\theta = \frac{\pi}{3}}$ $n = 0$

$$2\theta = \frac{4\pi}{3} + 2\pi n$$

$$\theta = \frac{1}{2}\left[\frac{4\pi}{3} + 2\pi n\right]$$

$n=0$: $\theta = \frac{1}{2}\left[\frac{4\pi}{3}\right] = \frac{2\pi}{3}$ ✓

$n=1$ $\theta = \frac{1}{2}\left[\frac{4\pi}{3} + 2\pi\right]$

$= \frac{1}{2}\left[\frac{10\pi}{3}\right]$

$= \frac{5\pi}{3}$ ✓

$$2\theta = \frac{2\pi}{3} + 2\pi n$$

$$\theta = \frac{1}{2}\left[\frac{2\pi}{3} + 2\pi n\right]$$

$$= \frac{\pi}{3} + \pi n.$$

$n=0$: $\theta = \frac{\pi}{3}$ ✓

$n=1$: $\theta = \frac{\pi}{3} + \pi$

$= \frac{4\pi}{3}$ ✓

$n=2$: $\theta = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$ ✗