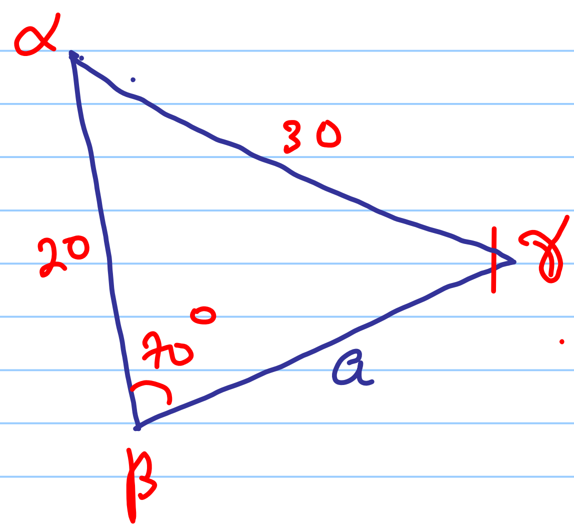
Note Title 12/2/2010

Example

Solve the triangle - $b = 30$, $c = 20$, $\beta = 70^\circ$.



Sine Law:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$\frac{\sin\alpha}{a} = \frac{\sin 70^\circ}{30} = \frac{\sin\gamma}{20}$$

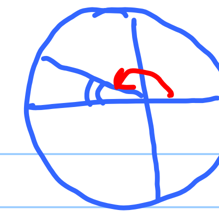
$$\frac{\sin\gamma}{20} = \frac{\sin 70^\circ}{30}$$

$$\sin\gamma = \frac{20 \sin 70^\circ}{30}$$

$$\approx 0.62646$$

$$\Rightarrow \gamma = \sin^{-1}(0.62646)$$

Sine is positive in Q I & Q II



$\Rightarrow \gamma \simeq 39^0$ in Q I

OR $\gamma = 141^\circ$

will not work.

Hence $\alpha = 180^\circ - (\beta + \gamma)$

$= 180^\circ - (70^\circ + 39^\circ)$

$= 71^\circ$

Now

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin 71^\circ}{a} = \frac{\sin 70^\circ}{30}$$

$$\Rightarrow a = \frac{30 \sin 71^\circ}{\sin 70^\circ}$$

$$a \approx 30$$

Example

Solve the triangle $a = 13$, $b = 26$, $\alpha = 120^\circ$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$\frac{\sin 120^\circ}{13} = \frac{\sin \beta}{26}$$

$$\sin\beta = \frac{26 \sin 120^\circ}{13}$$

$$\approx 1.73 > 1$$ Not possible

No such triangle.

8.2 The Law of Cosines

$$\boxed{\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}}$$

Law of Sines

For a triangle with sides $a, b,$ and c and opposite angles $\alpha, \beta,$ and γ, the following are true

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Example Given SAS (two sides and angle between them)

Solve the triangle: $a = 4.2$, $b = 7.3$ $\gamma = 25^\circ$

Step 1 Find c.

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$c^2 = (4.2)^2 + (7.3)^2 - 2(4.2)(7.3)\cos(25^\circ)$$

$$= 15.3552$$

$$c \approx \pm 3.9$$

C is positive

$$\boxed{C \approx +3.9}$$

<u>Step 2:</u> Find α

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\frac{\sin\alpha}{4.2} = \frac{\sin 25^\circ}{3.9} \Rightarrow \sin\alpha = \frac{4.2\sin 25^\circ}{3.9}$$

$$\alpha = \sin^{-1}\left(\frac{4.2 \sin 25^\circ}{3.9}\right)$$

$$\approx 27^\circ$$

Step 3: Find β

$$\beta = 180^\circ - (\alpha + \gamma)$$

$$= 180^\circ - (27^\circ + 25^\circ)$$

$$= 128^\circ$$

Example Given SSS

Solve the triangle $a=6$, $b=9$, $c=12$

S Step 1. Identify the largest angle, which is γ

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$12^2 = 6^2 + 9^2 - 2(6)(9)\cos\gamma$$

$$\cos\gamma = \frac{12^2 - 6^2 - 9^2}{-2(6)(9)}$$

$$\cos\gamma = \frac{27}{-108}$$

$$\gamma = \cos^{-1}\left(-\frac{27}{108}\right) = \cos^{-1}\left(-\frac{1}{4}\right)$$

$$\approx 104^{\circ}.$$

<u>Step 2.</u> Find either of remaining angles

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a}.$$

$$\frac{\sin 104^\circ}{12} = \frac{\sin\alpha}{6}$$

$$\sin\alpha = \frac{6\sin 104^\circ}{12}$$

$$\alpha = \sin^{-1}\left(\frac{6\sin 104^\circ}{12}\right)$$

$$\approx 29^\circ$$

$$\beta = 180^\circ - (29^\circ + 104^\circ) = 47^\circ$$