Note Title 12/1/2010

Questions

Example

$6\cos\theta + 1 = -2 \quad \text{on} \quad 0 \leq \theta < 2\pi$

$6\cos\theta = -3$

$\cos\theta = -\frac{1}{2}$

$\therefore \theta = \frac{2\pi}{3} \quad \text{or} \quad \theta = \frac{4\pi}{3}$

Example

Solve $2\sin^2\theta - \sin\theta - 1 = 0$ on $0 \leq \theta < 2\pi$

Let $u = \sin\theta$

$2u^2 - u - 1 = 0$ Quadratic

$(2u+1)(u-1) = 0$

either $2u+1 = 0$ or $u - 1 = 0$

$u = -\frac{1}{2}$ $u = 1$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \theta = \frac{11\pi}{6}$

$\sin\theta = 1$

$\theta = \frac{\pi}{2}$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

Example using Identities

Solve $\sin x = -\cos x$ on $0 \leq \theta < 2\pi$

$$\sin x = -\cos x$$

$$\frac{\sin x}{\cos x} = -\frac{\cos x}{\cos x}$$

$$\tan x = -1$$

tangent is negative in Q II, Q IV

$$\therefore \boxed{x = \frac{7\pi}{4}, \frac{3\pi}{4}}$$

Example

Solve $\sin x + \csc x = 2$ on $0 \le \theta < 2\pi$

$$\sin x + \csc x = 2$$

$$\sin x + \frac{1}{\sin x} = 2$$

Common denominator

$$\frac{\sin^2 x + 1}{\sin x} = 2$$

multiply both sides by $\sin x$.

$$\sin^2 x + 1 = 2\sin x$$

Quadratic

$$\sin^2 x - 2\sin x + 1 = 0$$

$$(\sin x - 1)(\sin x - 1) = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1 \quad \therefore \boxed{x = \frac{\pi}{2}}$$

8.1 Oblique Triangles and the Law of Sines

Example

Solve the triangle: $\beta = 75^\circ$, $\gamma = 60^\circ$, $b = 251\text{ m}$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha = 180^\circ - (75^\circ + 60^\circ)$$

$$= 45^\circ$$

Now need to determine a & c.

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

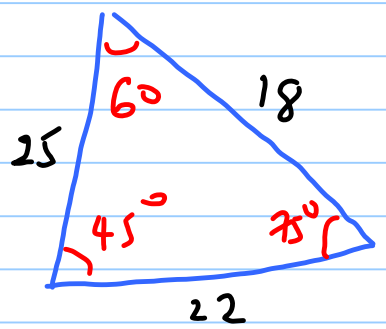
$$\boxed{\frac{\sin 45^\circ}{a} = \frac{\sin 75^\circ}{25} = \frac{\sin 60^\circ}{c}}$$

$$\frac{\sin 45^\circ}{a} = \frac{\sin 75^\circ}{25}$$

$$\Rightarrow a = \frac{25\ \sin 45^\circ}{\sin 75^\circ} \approx 18 \text{ in} \quad \left(\text{to two significant digits}\right)$$

$$\frac{\sin 60^\circ}{c} = \frac{\sin 75^\circ}{25}$$

$$c = \frac{25 \sin 60^\circ}{\sin 75^\circ} \approx 22 \text{ in}$$



Example

Solve the triangle $\gamma = 100°$, $\beta = 40°$, $a = 16$ ft

$$\alpha = 180° - (100° + 40°)$$

$$= 40°$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$\frac{\sin 40^\circ}{16} = \frac{\sin 40^\circ}{b} = \frac{\sin 100^\circ}{c}$$

Sine Law

$$\frac{\sin 40^\circ}{b} = \frac{\sin 40^\circ}{16} \Rightarrow \boxed{b = 16}$$

$$\frac{\sin 100^\circ}{c} = \frac{\sin 40^\circ}{16}$$

$$c = \frac{16 \sin 100^\circ}{\sin 40^\circ} \approx 25 \text{ ft}.$$