August 30, 2010

Note Title 8/30/2010

Example

Solve the inequality $\frac{x}{x+2} \leq 3$.

Solution

$$\frac{x}{x+2} \leq 3$$

$$\frac{x}{x+2} - 3 \leq 0$$

$$\frac{x}{x+2} - \frac{3(x+2)}{x+2} \leq 0$$

$$\frac{x - 3(x+2)}{x+2} \leq 0$$

$$\frac{-2x - 6}{x+2} \leq 0$$

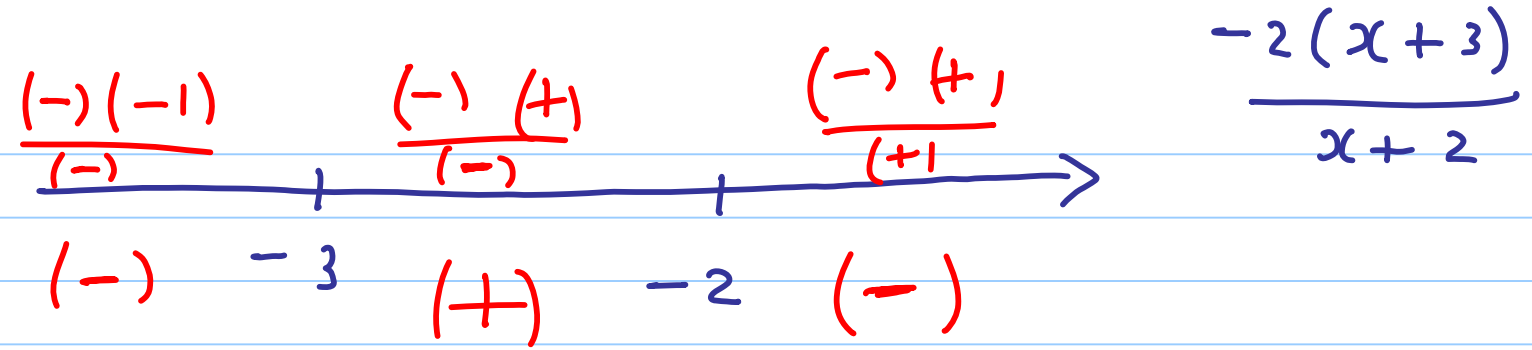
Zeros numerator $-2x - 6 = 0$

$-2x = 6$

$\boxed{x = -3}$

denominator $x + 2 = 0$

$x = -2$



$$\frac{-2x-6}{x+2} \leq 0$$

$$(-\infty, -3) \cup (-2, \infty)$$

Include the endpoint $x = -3$

$$(-\infty, -3] \cup (-2, \infty)$$

§1.6 #38

$$\frac{1-x}{x^2-9} \leq 0$$

Zeros: Numerator $1-x=0$

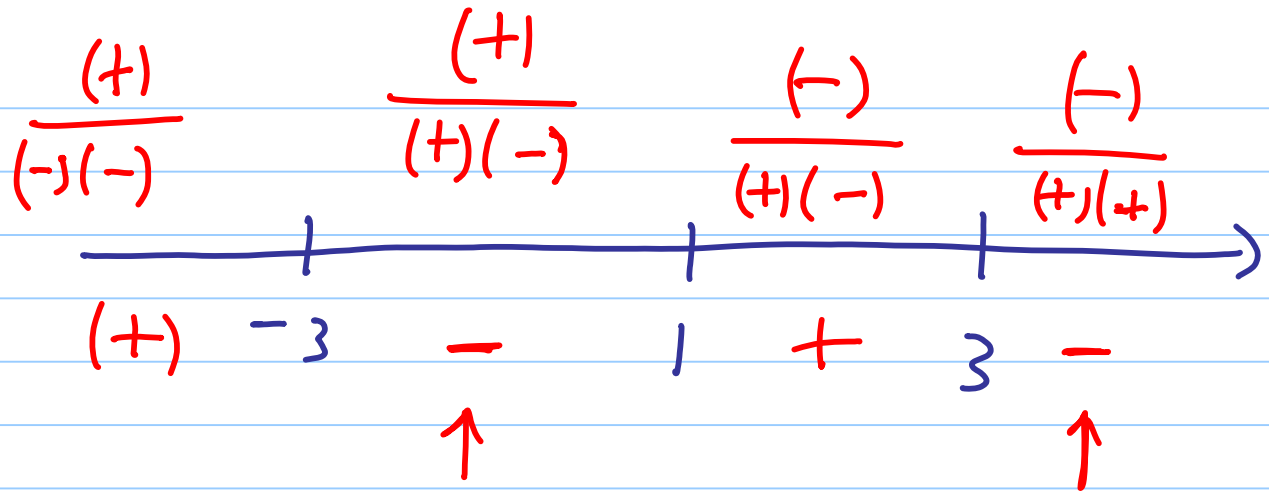
$1=x$

Denominator: $x^2-9=0$

$(x+3)(x-3)=0$

$x=\pm 3$

$$\frac{1-x}{x(x+3)(x-3)} \leq 0$$



$$(-3, 1) \cup (3, \infty)$$

End points? $x = \pm 3$

$$(-3, 1] \cup (3, \infty)$$

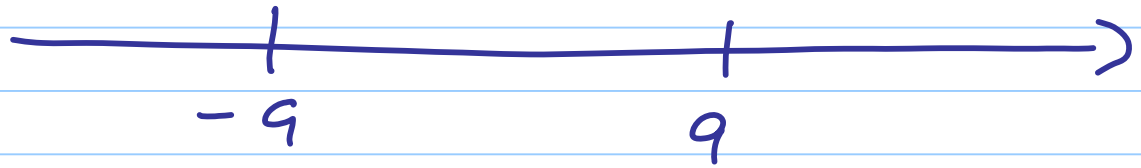
#23 $t^2 < 81$

$t^2 - 81 < 0$

zeros: $t^2 - 81 = 0$

$(t-9)(t+9) = 0$

$t = 9$ or $t = -9$



#15] $x^2 - 4x < 6$

$x^2 - 4x - 6 < 0$

Zeros: $x^2 - 4x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 24}}{2}$$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$$= 2 \pm \sqrt{10}$$

$x = 2 + \sqrt{10}$ or $x = 2 - \sqrt{10}$

1.7 Absolute value equations

Example

$|x-3| = 8$

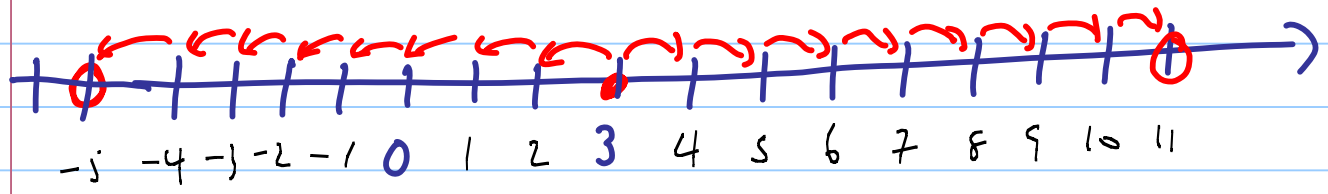
algebraically: $|x-3| = 8$

definition $x-3 = -8$ or $x-3 = 8$

$x = -5$ or $x = 11$

The solution set $\{-5, 11\}$.

graphically:



Example

$|1-3x| = 7$

either

$1-3x = -7$

$-3x = -8$

$x = 8/3$

or

$1-3x = 7$

$-3x = 6$

$x = -2$

$\{-2, 8/3\}$

Example

Solve $|1-3x| = -7$ absolute can never be negative

So NO SOLUTION

Example

$|5-x^2| = 1$

$5-x^2 = 1$ or $5-x^2 = -1$

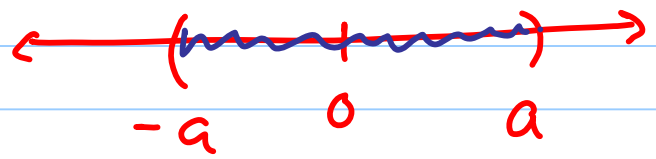
$-x^2 = -4$ $\quad -x^2 = -6$

$x^2 = 4$ $\quad x^2 = 6$

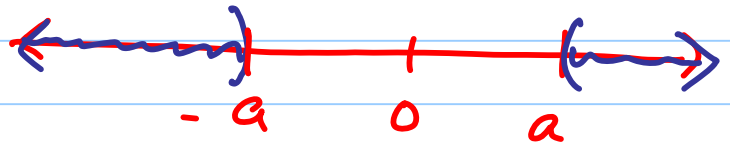
$x = \pm 2$ $\quad x = \pm\sqrt{6}$

Solution set $\{\pm 2, \pm\sqrt{6}\}$

$|x| < a$



$|x| > a$



Solve $|3x-2| \leq 7$

$|3x-2| \leq 7$

$-7 \leq 3x-2 \leq 7$

Add 2 to each part

$-5 \leq 3x \leq 9$

divide by 3

$\frac{-5}{3} \leq x \leq 3$

solution $\left[-\frac{5}{3}, 3\right]$

Example

$|1-2x| > 5$

$1-2x < -5$ or $1-2x > 5$

$-2x < -6$ $\qquad -2x > +4$

$\boxed{x > 3}$ $\qquad \boxed{x < -2}$

$(-\infty, -2) \cup (3, \infty)$