August 25, 2010

Note Title 8/25/2010

§1.3 #62)

$$4x^2 - 2x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-7)}}{2(4)}$$

$$= \frac{2 \pm \sqrt{4 + 112}}{8}$$

$$= \frac{2 \pm \sqrt{116}}{8}$$

$$= \frac{\cancel{2} \pm \cancel{2}\sqrt{29}}{\cancel{8}_4}$$

$$= \frac{1 \pm \sqrt{29}}{4}$$

$$\sqrt{116} = \sqrt{4 \cdot 29}$$
$$= \sqrt{4}\sqrt{29}$$
$$= 2\sqrt{29}$$

#50] $x^2 - 7x + 12 = 0$ Completing the square

$$x^2 - 7x = -12$$

$$x^2 - 7x + \left(\frac{-7}{2}\right)^2 = -12 + \left(\frac{-7}{2}\right)^2$$

$$\left(x - \frac{7}{2}\right)^2 = -12 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{-48 + 49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{1}{4}$$

$$x - \frac{7}{2} = \pm\sqrt{\frac{1}{4}}$$

$$x - \frac{7}{2} = \pm\frac{1}{2}$$

$$x = \frac{7}{2} \pm \frac{1}{2}$$

either $x = \frac{7}{2} + \frac{1}{2}$ $=$

or $x = \frac{7}{2} - \frac{1}{2}$ $=$

§1.4 Other Types of Equations

Example

$\sqrt{x-3} = 2$ $\qquad$ $\sqrt{a} = a^{1/2}$

Square both sides

$(\sqrt{x-3})^2 = 2^2$

$x - 3 = 4$ Linear

$x = 7$

Check: $\sqrt{7-3} = \sqrt{4} = 2$ ✓

The solution set is $\{7\}$

Example

$$\sqrt{2x+3} = x$$

Square both sides

$$\left(\sqrt{2x+3}\right)^2 = x^2$$

$$2x+3 = x^2$$

$$x^2 - 2x - 3 = 0$$ Quadratic equation

$$(x-3)(x+1) = 0$$

either $x-3=0$ or $x+1=0$

$x=3$ $x=-1$

Solutions to the quadratic equation are $\{3,-1\}$.

Are they also solutions to the radical equation?

Check!

$x=3$: $\sqrt{2x+3}=x$

$$\sqrt{2(3)+3} \stackrel{?}{=} 3$$

$$\sqrt{6+3} \stackrel{?}{=} 3$$

$$\sqrt{9} \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

$x = -1$: $\sqrt{2x+3} = x$

$$\sqrt{2(-1)+3} \stackrel{?}{=} -1$$

$$\sqrt{1} \stackrel{?}{=} -1$$

$$1 \stackrel{?}{=} -1 \quad \text{No!}$$

The solution set is $\{3\}$.

Example 3

$$\sqrt{x+2} + \sqrt{7x+2} = 6$$

$$\sqrt{7x+2} = 6 - \sqrt{x+2}$$

Squaring $$7x+2 = \left(6 - \sqrt{x+2}\right)^2$$

$$= \left(6 - \sqrt{x+2}\right)\left(6 - \sqrt{x+2}\right)$$

$$7x+2 = 36 - (2)6\sqrt{x+2} + (\sqrt{x+2})^2$$

$$7x + x = 36 - 12\sqrt{x+2} + x + 2$$

$$\frac{6x}{6} = \frac{36}{6} - \frac{12\sqrt{x+2}}{6}$$

$$x = 6 - 2\sqrt{x+2}$$

$$x + 2\sqrt{x+2} = 6$$

$$2\sqrt{x+2} = 6 - x$$

Square both sides

$$\left(2\sqrt{x+2}\right)^2 = (6-x)^2$$

$(6-x)(6-x)$

$$4(x+2) = 36 - 12x + x^2$$

$$4x + 8 = 36 - 12x + x^2$$

$$0 = 28 - 16x + x^2$$ Quadratic

$$(x-14)(x-2) = 0$$

$$x = 14 \text{ or } x = 2$$

Check: $x = 14$

$$\sqrt{x+2} + \sqrt{7x+2} = 6$$

$$\sqrt{14+2} + \sqrt{7(14)+2} \stackrel{?}{=} 6$$

$$\sqrt{16} + \sqrt{100} \stackrel{?}{=} 6 \quad \text{NO}$$

$x = 2$:

$$\sqrt{x+2} + \sqrt{7x+2} = 6$$

$$\sqrt{2+2} + \sqrt{7(2)+2} \stackrel{?}{=} 6$$

$$\sqrt{4} + \sqrt{16} \stackrel{?}{=} 6 \quad \text{YES}$$

Solution $\{2\}$.

U substitution

Example

$$x^{-2} - x^{-1} - 12 = 0$$

$$(x^{-1})^2 - (x^{-1})^1 - 12 = 0$$

Let $u = x^{-1}$

$$u^2 - u - 12 = 0$$

$$(u-4)(u+3) = 0$$

either $u = 4$ or $u = -3$

$x^{-1} = 4$

$\frac{1}{x} = 4$

$x = \frac{1}{4}$

$x^{-1} = -3$

$\frac{1}{x} = -3$

$x = -\frac{1}{3}$

Check!