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§1.3 #19

$4x = 4x^2 + 1$

$4x^2 - 4x + 1 = 0$

$(2x - 1)(2x - 1) = 0$

$4x^2$
$\quad - 2x - 2x + 1$

$2x - 1 = 0 \quad$ or $\quad 2x - 1 = 0$

$2x = 1 \qquad\qquad\qquad x = \frac{1}{2}$

$x = \frac{1}{2}$

Factor of 1

Factor of $4x^2$

$4x \quad x$
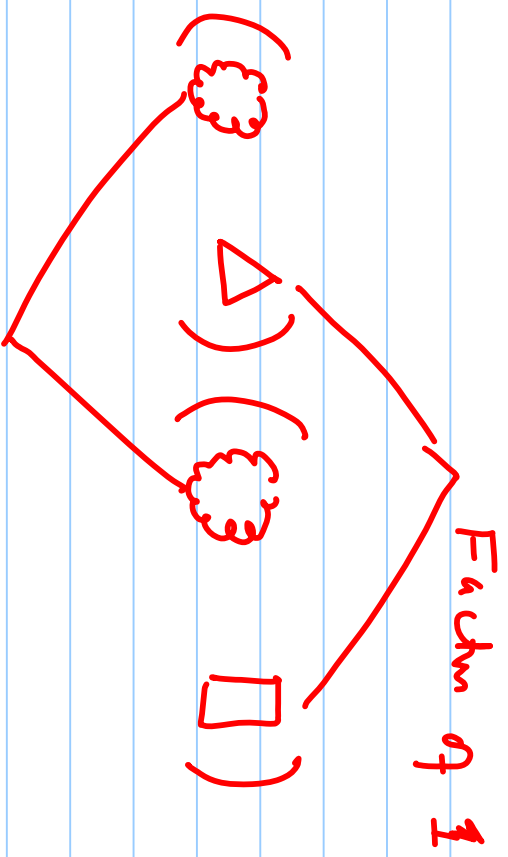
$2x \quad 2x$

Completing the square method

Example 7

$$x^2 + 8x - 3 = 0$$

$$x^2 + 8x = 3 \quad \text{(Step 1)}$$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = 3 + \left(\frac{8}{2}\right)^2 \quad \text{(Step 2)}$$

$$\underbrace{x^2 + 8x + (4)^2}_{\text{perfect square!}} = 3 + 16$$

$$(x + 4)^2 = 19$$

Step 3 Square root

$$x + 4 = \pm\sqrt{19}$$

$$x = -4 \pm \sqrt{19}$$

either

$$x = -4 + \sqrt{19} \qquad \sim \qquad x = -4 - \sqrt{19}$$

Example 8

$$3x^2 - 12x + 13 = 0$$

$$3x^2 - 12x = -13$$

Divide both side by 3

$$x^2 - \frac{12}{3}x = -\frac{13}{3}$$

$$x^2 - 4x = -\frac{13}{3}$$

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = -\frac{13}{3} + \left(\frac{-4}{2}\right)^2$$

$$x^2 - 4x + (-2)^2 = \frac{-13}{3} + 4$$

$$(x-2)^2 = -\frac{1}{3}$$

Take square roots

$$x - 2 = \pm\sqrt{\frac{-1}{3}}$$

$$x = 2 \pm \sqrt{-\frac{1}{3}}$$

$$= 2 \pm \sqrt{\frac{1}{3}}\, i$$

Example

$$x^2 - 4x - 1 = 0$$

$$x^2 - 4x = 1$$

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = 1 + \left(\frac{-4}{2}\right)^2$$

$$x^2 - 4x + (-2)^2 = 1 + 4$$

$$(x-2)^2 = 5$$

$$x - 2 = \pm\sqrt{5} \implies x = 2 \pm \sqrt{5}$$

Quadratic formula:

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{divide both sides by } a.$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example

$$x^2 - 4x - 1 = 0$$

$$a = 1, \; b = -4, \; c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2}$$

$$\sqrt{20} = \sqrt{4}\sqrt{5}$$
$$= 2\sqrt{5}$$

$$\boxed{x = 2 \pm \sqrt{5}}$$

Example

$$x^2 + 8 = 4x$$

$$x^2 - 4x + 8 = 0$$

$$a = 1, b = -4, c = 8$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-32}}{2}$$

$$= \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm 4i}{2}$$

$$\boxed{x = 2 \pm 2i}$$

Example

$4x^2 - 4x + 1 = 0$

$a = 4, \quad b = -4, \quad c = 1$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{4 \pm 0}{8} = \frac{1}{2}$$