MATHEMATICAL CONCEPTS - MAT 117

SPRING 2009 - REVIEW 4

No. I. Use the graph below to answer the following questions.

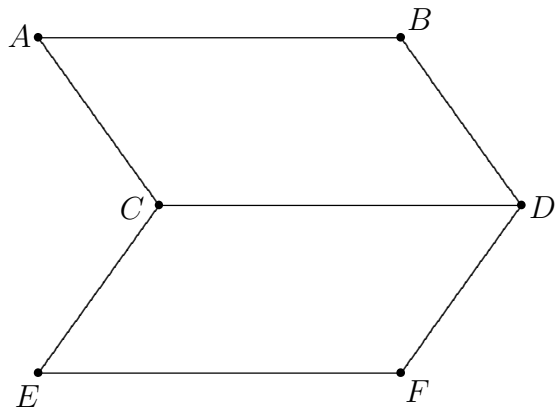


Figure 1:

a) Is A, B, C, D, F, E a path? Explain.

NO. There is no edge connecting vertices B and C.

b) Which edge(s) shown on the graph are not included in the following path: A, B, D, F, E, C?

edge AC and edge CD.

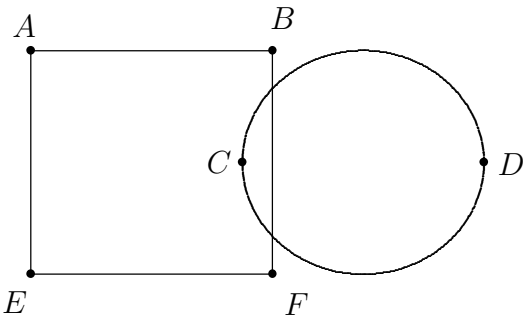
c) On the graph, is it possible to determine a circuit that includes only four distinct vertices? If so, determine one such circuit.

YES. A, B, D, C, A

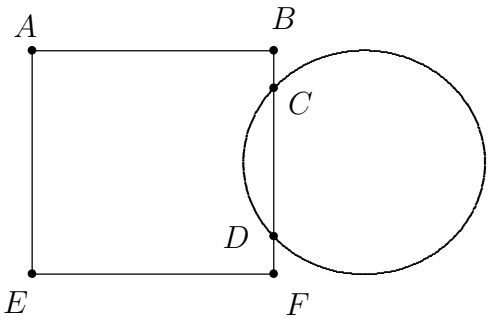
or C, D, F, E, C

etc.

No. II. Determine whether each of the graphs shown is connected or disconnected.



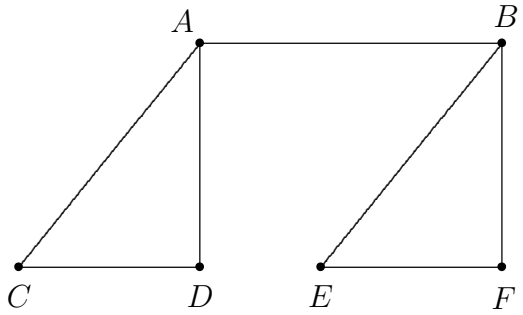
Disconnected



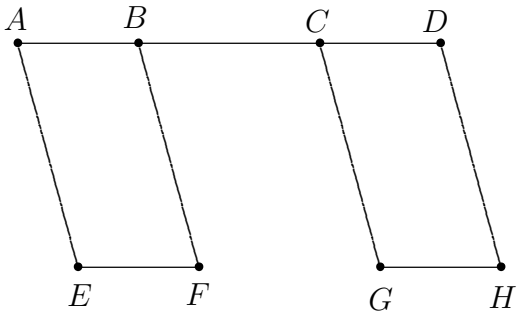
Connected

Figure 2:

No. III. Determine whether each of the connected graphs shown has a bridge or not.



Edge AB



Edge BC

Figure 3:

No. IV. Use the graph below (Figure 4) to answer the following questions.

(a) Determine an Euler path that begins with vertex A.

A, B, A, C, B, E, C, D, A, D, E etc

(b) Determine an Euler path that begins with vertex E.

E, D, A, B, E, C, D, A, B, C, A

(c) Is it possible to determine an Euler circuit for this graph? Explain.

NO. A graph with exactly two odd vertices has no Euler circuit.

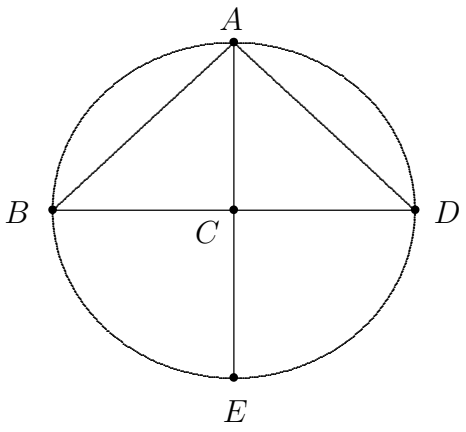
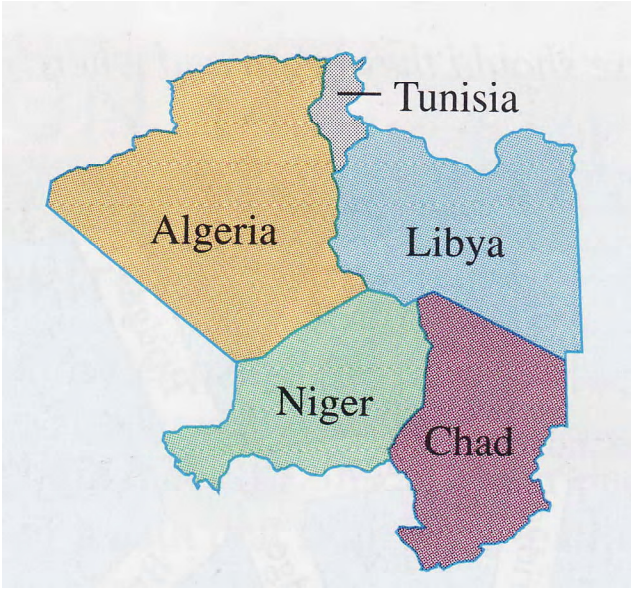


Figure 4:

(d) Is it possible to determine an Euler path that begins with vertex C? Explain.

NO. This graph has exactly two odd vertices. Each Euler path must begin with an odd vertex. C is an even vertex.

No. V. Consider the map given in Figure 5, representing a part of Northern Africa. Algeria (A), Chad (C), Libya (L), Niger (N), and Tunisia (T).



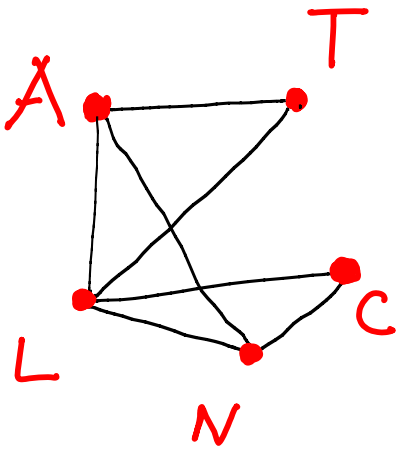


Figure 5:

- Represent the map as a graph. Use the letter indicated in parenthesis to label vertices of the graph.

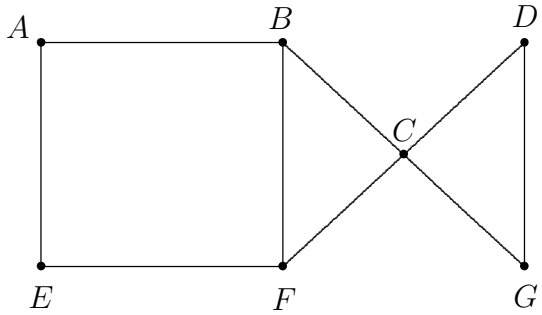
- Determine (state yes or no) whether the graph in part (a) has an Euler path. If yes, give one such Euler path.

YES: A, T, L, C, N, L, A, N

- Determine (state yes or no) whether the graph in part (a) has an Euler circuit. If yes, give one such Euler circuit.

NO

No. VI. Determine two different Hamilton paths in the graph below (Figure 6).

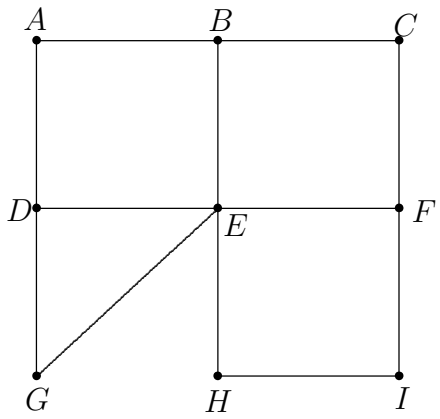


A, E, F, B, C, D, G

G, D, C, F, E, A, B

Figure 6:

No. VII. Determine two different Hamilton circuits in the graph below (Figure 7).



A, B, C, F, I, H, E, G, D, A

E, H, I, F, C, B, A, D, G, E

Figure 7:

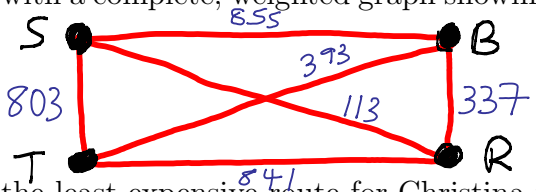
No. VIII. Rita Hindernan lives in Sumter, South Carolina, and wishes to visit relatives in the following South Carolina cities: Anderson, Charleston, Columbia, Florence, Greenville, Rock Hill and Spartanburg. In how many ways can she visit each of these cities and return to her home in Sumter?

$n = 8$

$(8-1)! = 7! = 5040$ ways

No. IX. Christina Dweyer is searching for a new job. She lives in Shreveport, Louisiana, and has interviews in Barrow, Alaska; Tucson, Arizona; and Rochester, New York. The costs of the one-way flights between these four cities are as follows: Shreveport to Barrow costs \$855, Shreveport to Tucson costs \$803, Shreveport to Rochester costs \$113, Barrow to Tucson costs \$393, Barrow to Rochester costs \$337, and Tucson to Rochester costs \$841.

(a) Represent this traveling salesman problem with a complete, weighted graph showing the prices of the flights on the appropriate edges.



(b) Use the Brute force method to determine the least expensive route for Christina to travel to each city and return home to Shreveport.

STRBS $803 + 841 + 337 + 855 = 2836$

STBRS $803 + 393 + 337 + 113 = 1646$

(c) What is the minimum cost she can pay?

SBTRS $855 + 393 + 841 + 113 = 2202$

SBRTS $855 + 337 + 841 + 803 = 2836$

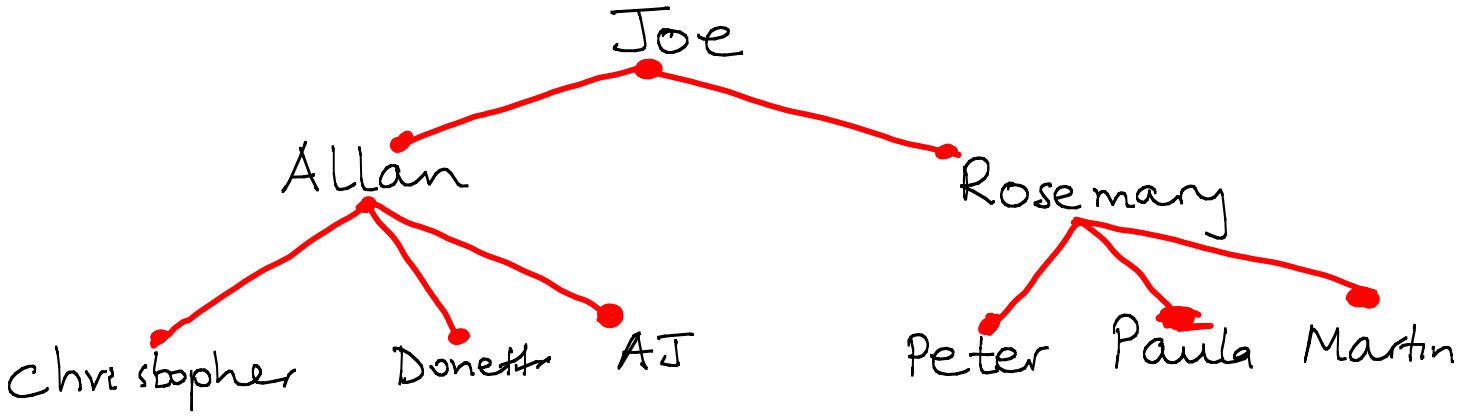
SRBTS $113 + 337 + 393 + 803 = 1646$

SRTBS $113 + 841 + 393 + 855 = 2202$

\$1646

STBRS or SRBTS List cities!

No. X. Use a tree to show the parent-child relationships in the following family. Joe has two children: Allan and Rosemary. Allan has three children: Christopher, Donetta and AJ. Rosemary has three children: Peter, Paula and Martin.



No. XI. Determine two different spanning trees for the given graph.

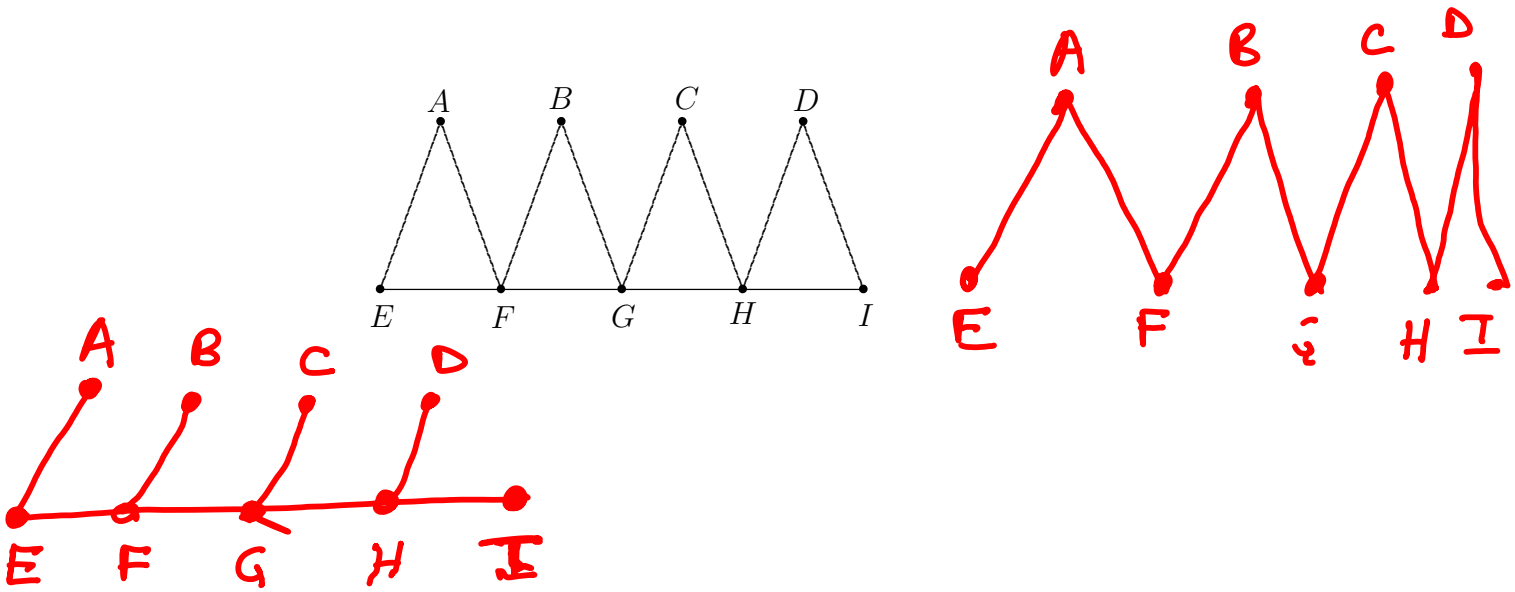


Figure 8:

No. XII. Determine the minimum-cost spanning trees for the given graphs.

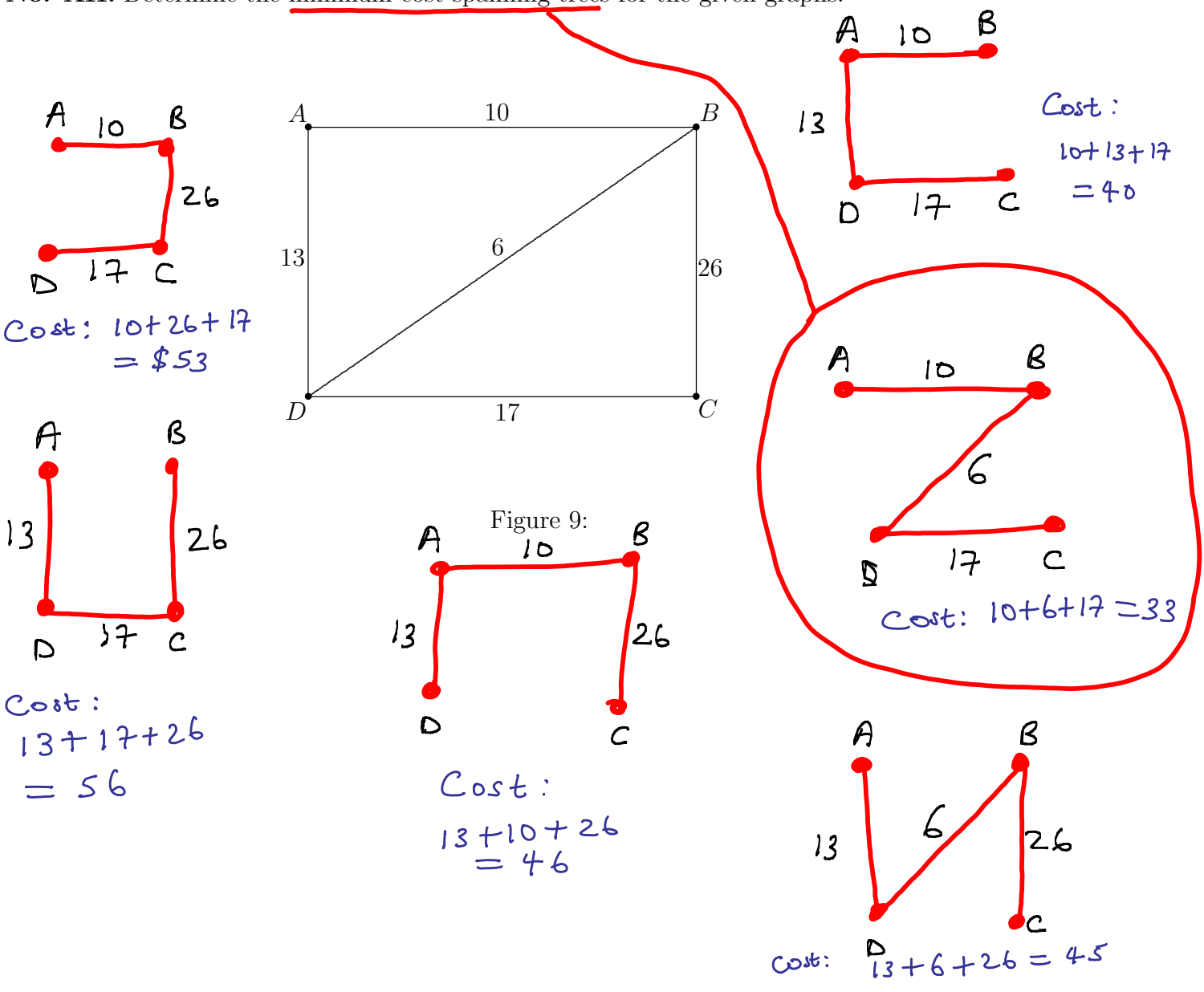


Figure 9: