13.5 MEASURES OF CENTRAL TENDENCY

Mathematical Concepts

April 2, 2009

An **average** is a number that is representative of a group of data. There are at least four different averages

- the mean
- the median
- the mode
- the midrange

Each average is calculated differently and may yield different results for the same set of data.
Averages are commonly referred to as **measures of central tendency** because each will result in a number near the center of the data.

The **arithmetic mean** or simply the **mean** is symbolized either by $\bar{x}$ (read "x bar") or by the Greek letter mu, μ.
The symbol $\bar{x}$ is used when the mean of a *sample* of the population is calculated.
The symbol μ is used when the mean of the *entire population* is calculated.

The Greek letter sigma, Σ, is used to indicate "summation."
The notation Σx, read "the sum of x," is used to indicate the sum of all the data. For example, if there are five pieces of data, 4, 6, 1, 0, 5, then $\Sigma x = 4 + 6 + 1 + 0 + 5 = 16$.

The **mean**, $\bar{x}$ is the sum of the data divided by the number of pieces of data. The formula for calculating the mean is

$$\bar{x} = \frac{\Sigma x}{n} \tag{1}$$

where Σx represents the sum of all the data and n represents the number of pieces of data.

Determine the mean age of a group of patients at a doctor's office if the ages of the individuals are 27, 18, 48 34, and 48.

solution:

$$\bar{x} = \frac{\Sigma x}{n} = \frac{27 + 18 + 48 + 34 + 48}{5} = \frac{175}{5} = 35$$

Therefore, the mean, $\bar{x}$, is 35 years.

The mean represents "the balancing point" of a set of data.

To find the median of a set of data,

- *rank the data* from the smallest to largest, or largest to smallest
- determine the value in the middle of the set of *ranked data*

The **median** is the value in the middle of a set of *ranked data*.

Determine the median of the ages of a group of patients at a doctor's office if the ages of the individuals are 27, 18, 48, 34, and 48.

solution:
Ranking the data from smallest to largest gives

$$18, \quad 27, \quad 34, \quad 48, \quad 48.$$

Since 34 is the value in the middle of this set of ranked data, 34 years is the median age.

Determine the median of the following sets of data.

a) 9, 14, 16, 17, 11, 16, 11, 12

Ranking the data gives

$$9, \quad 11, \quad 11, \quad 12, \quad 14, \quad 16, \quad 16, \quad 17.$$

There are eight pieces of data. Therefore, the median will lie halfway between the two middle pieces, the 12 and the 14. The median is $\frac{12+14}{2} = \frac{26}{2} = 13$.

b) 7, 8, 8, 8, 9, 10

There are six pieces of data, and they are already ranked. Therefore, the median lies halfway between the two middle pieces. Both middle pieces are 8's.
The median is $\frac{8+8}{2} = \frac{16}{2} = 8$.

The **mode** is the piece of data that occurs most frequently.

- If each piece of data occurs only once, the set of data has no mode.
- If two values in a set of data occur more often than all the other data, we consider both these values as modes and say that the data are **bimodal**.

Determine the mode of the ages of a group of patients at a doctor's office if the ages of the individuals are 27, 18, 48, 34, and 48.

solution:
The age 48 is the mode because it occurs twice and the other values occur only once.

The **midrange** is the value halfway between the lowest (L) and the highest (H) values in a set of data.

A formula for finding the midrange is

$$\textit{Midrange} = \frac{\text{lowest value} + \text{highest value}}{2}. \tag{2}$$

Determine the midrange of the ages of a group of patients at a doctor's office if the ages of the individuals are 27, 18, 48, 34, and 48.

solution:

The lowest age is 18, and the highest age is 48.

$$\textit{Midrange} = \frac{\text{lowest} \ + \text{highest}}{2} = \frac{18 + 48}{2} = \frac{66}{2} = 33.$$

The midrange is 33 years.

The salaries of eight selected social workers rounded to the nearest thousand dollars are 40, 25, 28, 35, 42, 60, 60, and 73. For this set of data, determine the

a) mean
b) median
c) mode
d) midrange
e) list the measures of central tendency from lowest to highest.

a) $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{40 + 25 + 28 + 35 + 42 + 60 + 60 + 73}{8} = \dfrac{363}{8}$,
that is, $\bar{x} = 45.375$.

b) Ranking the data from the smallest to the largest gives

$$25, \quad 28, \quad 35, \quad 40, \quad 42, \quad 60. \quad 60, \quad 73$$

Since there are an even number of pieces of data, the median is halfway between 40 and 42.

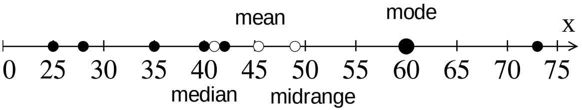
The median $= \frac{40+42}{2} = \frac{82}{2} = 41$.

c) The mode is the piece of data that occurs most frequently. The mode is 60.

d) The midrange $= \frac{L+H}{2} = \frac{25+73}{2} = \frac{98}{2} = 49$.

e) The averages from lowest to highest are the *median, mean, midrange* and *mode*. Their values are 41, 45.375, 45, and 60, respectively.

Which average do you feel is most representative of the salaries?


mean
mode
x
0
25
30
35
40
45
50
55
60
65
70
75
median
midrange

Measures of position are used to describe the position of a piece of data in relation to the rest of the data.

Measures of position are often used to make comparisons, such as comparing the scores of individuals from different populations, and are generally used when the amount of data is large.

Two measures of position are

- percentiles
- quartiles

There are 99 percentiles dividing a set of data into 100 equal parts.

For example, if you scored 520 on the math portion of the SAT, and the score of 520 was reported to be in the 78th percentile of high school students.

This wording *does not* mean that 78% of your answers were correct.

It *does* mean that you outperformed about 78% of all those taking the exam.

Quartiles divide data into four equal parts.

The first quartile is the value that is higher than about $\frac{1}{4}$ or 25% of the population. It is the same as the 25th percentile.

The second quartile is the value that is higher than about $\frac{1}{2}$ the population and is the same as the 50th percentile, or the median.

The third quartile is the value that is higher than about $\frac{3}{4}$ of the population and is the same as the 75th percentile.