

12.7 CONDITIONAL PROBABILITY

Mathematical Concepts

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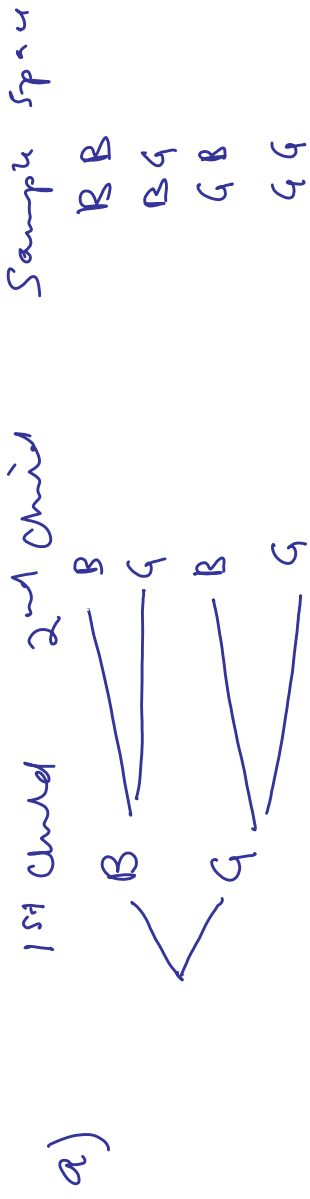
Definition:

The probability of event E_2 occurring, given that an event E_1 has happened (or will happen), is called a **conditional probability** and is written $P(E_2|E_1)$.

A single card is selected from a deck of cards. Determine the probability it is a club, give that it is black.

- ▶ The card is black. Only 26 cards are possible.
- ▶ 13 of the possible cards are clubs.
- ▶ So

$$P(\text{club} \mid \text{black}) = \frac{13}{26} = \frac{1}{2}.$$



Outcome: {GG}

$$P(2 \text{ girls}) = \frac{1}{4}$$

A family has two children. Assuming that boys and girls are equally likely, determine the probability that the family has

- (a) two girls.
- (b) two girls if you know that at least one of the children is a girl.
- (c) two girls given that the older child is a girl.

b) Sample space {BG, GB, GG}

$$P(\text{both girls} | \text{at least one is a girl}) = \frac{1}{3}$$

c) Sample space {GB, GG}

$$P(\text{both girls} | \text{older child is girl}) = \frac{1}{2}$$

CONDITIONAL PROBABILITY FORMULA

For any two events, E_1 and E_2 ,

$$P(E_2|E_1) = \frac{n(E_1 \text{ and } E_2)}{n(E_1)}$$

where

- ▶ $n(E_1 \text{ and } E_2)$ represents the number of sample points common to both event 1 and event 2
- ▶ $n(E_1)$ is the number of sample points in event E_1 .

Two hundred patients who either had hip surgery or knee surgery were asked whether they were satisfied, dissatisfied, or neutral regarding the results of their surgery. The responses are given in the table below.

Surgery	Satisfied	Dissatisfied	Total
Knee	70	25	95
Hip	90	15	105
Total	160	40	200

9) Total # of patients = 200

Number satisfied with results of the surgery = 160

$$P(\text{Satisfied with the results of the surgery}) = \frac{160}{200} = \frac{4}{5}$$

b) Let E_1 be the event "the person had knee surgery

event E_2 : the person was satisfied with the results of the surgery

$$P(E_2 | E_1) = \frac{n(E_1 \text{ and } E_2)}{n(E_1)} = \frac{70}{95}$$

If one person from the 200 patients surveyed is selected at random, determine the probability that the person

- was satisfied with the results of the surgery.
- was satisfied with the results of the surgery, given that the person had knee surgery.
- was dissatisfied with the results of the surgery, given that the person had hip surgery.
- had hip surgery, given that the person was dissatisfied with the results of the surgery.

c) event E_1 : the person had hip surgery
event E_2 : the person was dissatisfied with the results

$$P(E_2 | E_1) = \frac{n(E_1 \text{ and } E_2)}{n(E_1)} = \frac{15}{105} = \frac{1}{7}$$

d) event E_1 : the person was dissatisfied with the results

event E_2 : the person had hip surgery

$$P(E_2 | E_1) = \frac{n(E_1 \text{ and } E_2)}{n(E_1)} = \frac{15}{40} = \frac{3}{8}$$