12.6 *OR* AND *AND* PROBLEMS

Mathematical Concepts

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The **Or Probability problem** requires obtaining a "successful" outcome for *at least one* of the given events.

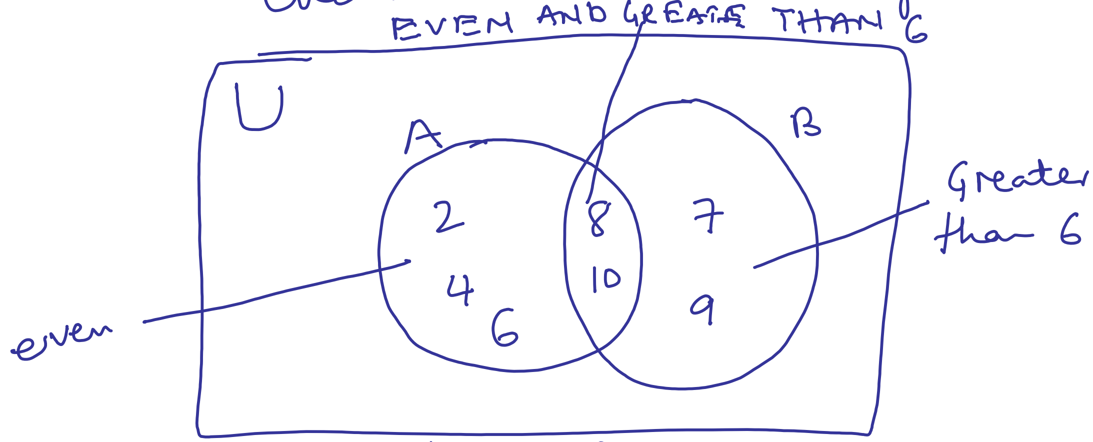
The probability of event A or event B is given by the formula

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Example 1

event A : the number is even

event B : the number is greater than 6



VENN DIAGRAM

Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a hat, and one piece is randomly selected. Determine the probability that the piece of paper selected contains an even number or a number greater than 6.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P\left(\begin{matrix}\text{even or}\\ \text{greater than } 6\end{matrix}\right) = P(\text{even}) + P\left(\begin{matrix}\text{greater}\\ \text{than } 6\end{matrix}\right) - P\left(\begin{matrix}\text{even and}\\ \text{greater than}\\ 6\end{matrix}\right)$$

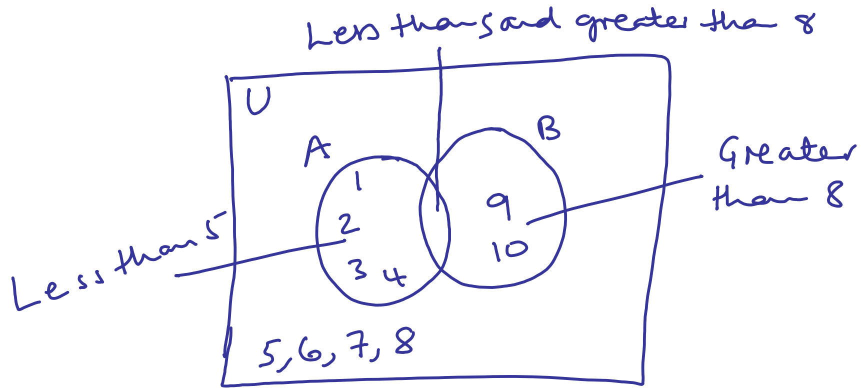
$$= \frac{5}{10} + \frac{4}{10} - \frac{2}{10}$$

$$= \boxed{\frac{7}{10}}$$

Example 2

event A : the number is less than 5

event B : the number is greater than 8.



Consider the same sample space, the numbers 1 through 10, as in the preceding example 1. If one piece of paper is selected, determine the probability that it contains a number less than 5 or greater than 8.

$$P(\text{number is less than } 5) = \frac{4}{10}$$

$$P(\text{number is greater than } 8) = \frac{2}{10}$$

$$P(\text{number is less than } 5 \text{ \& greater than } 8) = 0$$

$$P\left(\begin{array}{c}\text{number is} \\ \text{less than } 5 \\ \text{or greater} \\ \text{than } 8\end{array}\right) = \frac{4}{10} + \frac{2}{10} - 0$$

$$= \frac{6}{10} = \boxed{\frac{3}{10}}$$

Two events A and B are **mutually exclusive** if it is impossible for both event to occur simultaneously.

In this case, $P(A \text{ and } B) = 0$ and the addition formula simplifies to

$$P(A \text{ or } B) = P(A) + P(B).$$

Example 3

One card is selected from a standard deck of playing cards. Determine whether the following pairs of events are mutually exclusive and find $P(A \text{ or } B)$.

a) $A =$ an ace, $B =$ a jack

b) $A =$ an ace, $B =$ a heart

c) $A =$ a red card, $B =$ a black card

d) $A =$ a picture card, $B =$ a red card

a) Impossible to select both an ace and a jack when only one card is selected. ∴ Events are mutually exclusive

$$P(\text{ace or jack}) = P(\text{ace}) + P(\text{jack}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}.$$

b) One card, the ace of hearts, is both an ace and a heart. Therefore, these events are not mutually exclusive.

$$P(\text{ace or heart}) = P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$$
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$$

c) Events mutually exclusive because it is impossible to select one card that is both a red card and a black card.

$$P(\text{red or black}) = P(\text{red}) + P(\text{black})$$
$$= \frac{26}{52} + \frac{26}{52} = \frac{52}{52} = \boxed{1}$$

A red card or a black card must be selected.

d) NOT Mutually exclusive.

$$P\left(\begin{matrix}\text{picture card}\\ \text{or red card}\end{matrix}\right) = P(\text{picture card}) + P\left(\begin{matrix}\text{red}\\ \text{card}\end{matrix}\right) - P\left(\begin{matrix}\text{picture card}\\ \text{and red card}\end{matrix}\right)$$
$$= \frac{12}{52} + \frac{26}{52} - \frac{6}{52}$$
$$= \frac{32}{52} = \boxed{\frac{8}{13}}$$

The **and probability problem** requires obtaining a favorable outcome in *each* of the given events.

A formula for finding the probability of events A and B is

$$P(A \text{ and } B) = P(A) \cdot P(B).$$

Example 4

event A : selection of first queen

event B: selection of second queen.

$$P(A) = \frac{4}{52}$$

after first draw, the card is returned to the deck.

$$P(B) = \frac{4}{52}$$

Two cards are to be selected *with replacement* from a deck of cards. Determine the probability that two queens will be selected.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$\begin{aligned} P(2 \text{ queens}) &= P(\text{queen } 1 \text{ and queen } 2) \\ &= P(\text{queen } 1) \cdot P(\text{queen } 2) \\ &= \frac{4}{52} \cdot \frac{4}{52} \\ &= \frac{1}{13} \cdot \frac{1}{13} \\ &= \boxed{\frac{1}{169}} \end{aligned}$$

Example 5

$$P(\text{selecting first queen}) = \frac{4}{52}$$

after selecting first queen only 3 queens are left and the total number of cards is 51.

$$P(\text{selecting second queen}) = \frac{3}{51}$$

Two cards are to be selected *without replacement* from a deck of cards. Determine the probability that two queens will be selected.

$$P(2 \text{ queens}) = P(\text{queen } 1) \cdot P(\text{queen } 2)$$

$$= \frac{4}{52} \cdot \frac{3}{51}$$

$$= \frac{1}{13} \cdot \frac{1}{17}$$

$$= \boxed{\frac{1}{221}}$$

Event A and event B are **independent events** if the occurrence of either event in no way affects the probability of occurrence of the other event.