12.1 TREE DIAGRAMS)

Mathematical Concepts

March 3, 2009

COUNTING PRINCIPLE

If a first experiment can be performed in M distinct ways and a second experiment can be performed in N distinct ways, then the two experiments in that specific order can be performed in $M \cdot N$ distinct ways.

For example, consider tossing a coin and then rolling a die. The goal is to determine the number of possible outcomes.

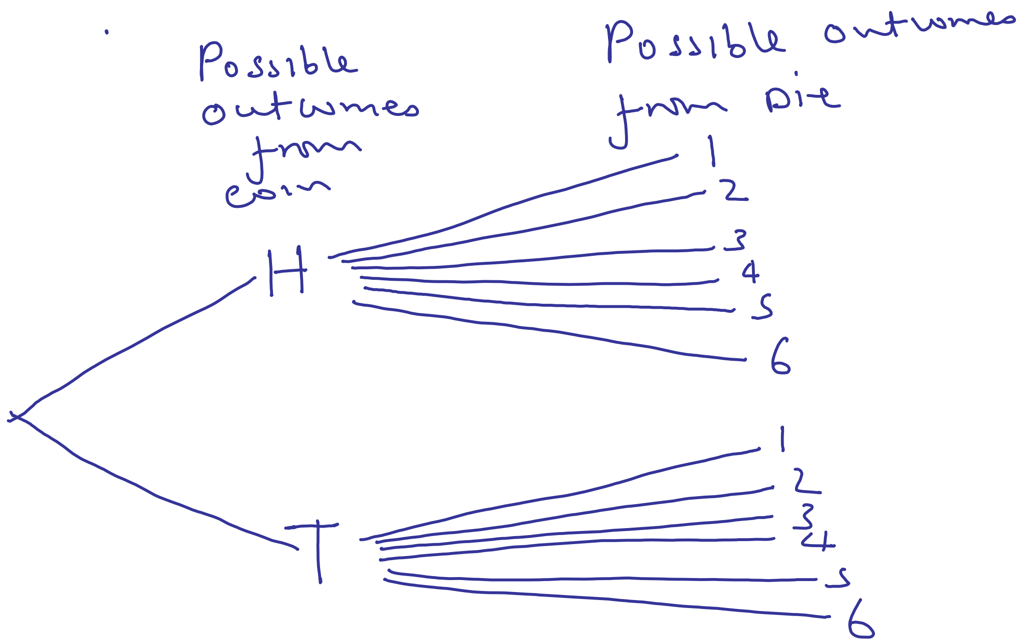
The coin has two possible outcomes, *heads* and *tails*. The die has six possible outcomes: *1*, *2*, *3*, *4*, *5*, and *6*. So the two experiments together have $2 \cdot 6 = 12$ possible outcomes.

Definitions:

- A list of all the possible outcomes of an experiment is called a **sample space**.
- Each individual outcome in the sample space is called a **sample point**.

Tree diagrams are helpful in determining sample spaces.

- A tree diagram illustrating **all** the possible outcomes when a coin is tossed and a die is rolled has two initial branches, one for each possible outcome of the coin.
- each of these branches will have six branches emerging from them, one for each possible outcome of the die.
- The sample space can be obtained by listing **all** the possible combinations of branches.



SAMPLE SPACE

H1
H2
H3
H4
H5
H6

T1
T2
T3
T4
T5
T6

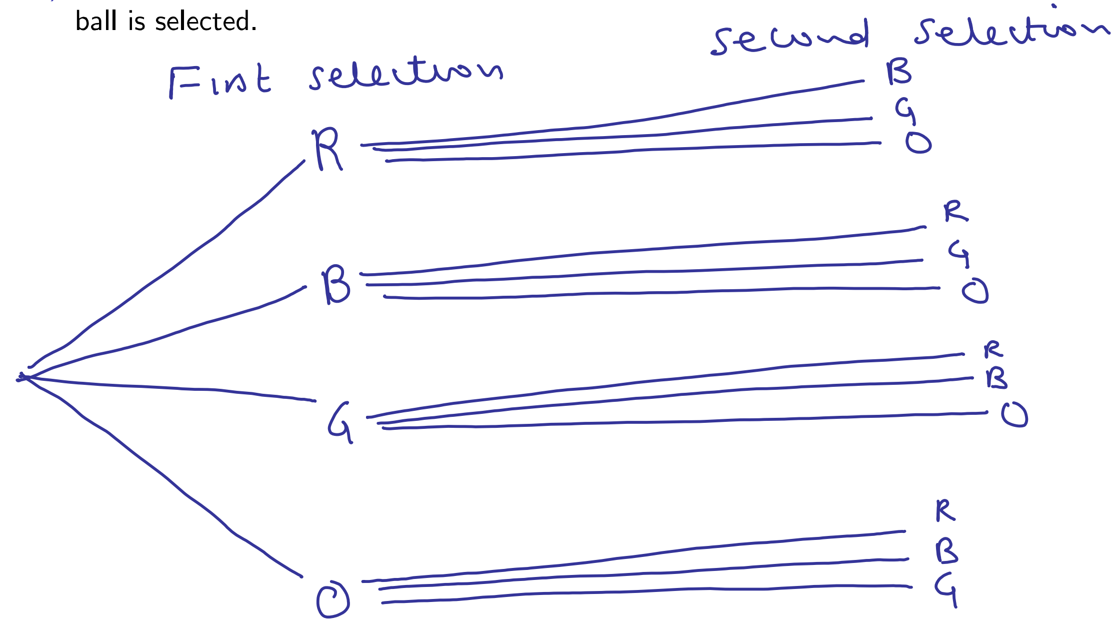
a) First selection: One of four balls
Second selection: only three balls remain
# of sample points is $4 \cdot 3 = 12$

$$P(E) = \frac{\#\text{ of outcomes favorable to } E}{\text{total number of outcomes}}$$

c) P(orange ball is selected) $= \frac{6}{12} = \frac{1}{2}$
d) P(green followed by red) $= 1/12$

Two balls are to be selected without replacement from a bag that contains one red, one blue, one green and one orange ball.

a) Use the counting principle to determine the number of points in the sample space.
b) Construct a tree diagram and list the sample space.
c) Determine the probability that one orange ball is selected.
d) Determine the probability that a green ball followed by a red ball is selected.

b)


SAMPLE SPACE
RB
RG
RO

BR
BG
BO

GR
GB
GO

OR
OB
OG

a) 2 choices for a sandwich

3 Choices for a beverage

2 Choices for desert

# of different lunch specials is $2 \cdot 3 \cdot 2 = 12$

c) P(roast beef and ice cream are selected) $= \frac{3}{12} = \frac{1}{4}$

d) P(neither tea nor ice cream are selected) $= \frac{4}{12} = \frac{1}{3}$

At Theresa's Restaurant each lunch special consists of a sandwich, a beverage, and a dessert. The sandwich choices are roast beef (r) or ham (h). The beverage choices are coffee (c), tea (t), or soda (s). The dessert choices are ice cream (i) or apple pie (p).

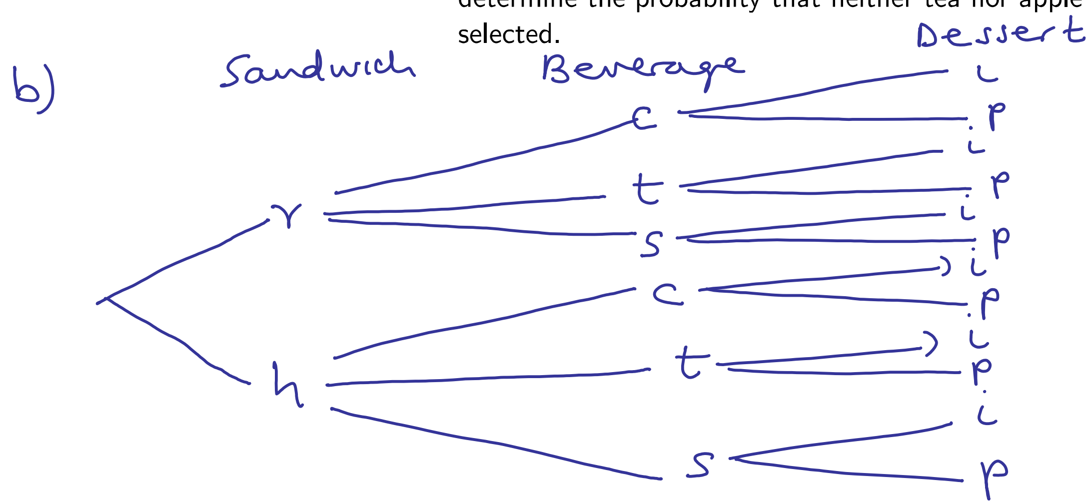
a) Use the counting principle to determine the number of different lunch specials offered by the restaurant.

b) Construct a tree diagram and list the sample space.

c) If a customer randomly selects one of the lunch specials, determine the probability that both a roast beef sandwich and ice cream are selected.

d) If a customer randomly selects one of the lunch specials, determine the probability that neither tea nor apple pie is selected.

b)



SAMPLE SPACE

rci
rcp
rti
rtp
rsi
rsp
hci
hcp
hti
htp
hsi
hsp

A radio station has two tickets to give to a Beyoncé concert. It held a contest and narrowed the possible recipients to four people: Christine (C), Mike Hammer (MH), Mike Levine (ML), and Phyllis (P). The names of two of these four people will be selected at random from a hat and the two people selected will be awarded the tickets.

a) Use the counting principle to determine the number of points in the sample space.

b) Construct a tree diagram and list the sample space.

c) Determine the probability that Christine is selected.

d) Determine the probability that neither Mike Hammer nor Mike Levine is selected.

e) Determine the probability that at least one Mike is selected.

a) 1st selection: any of the four people

2nd selection: only three people remain

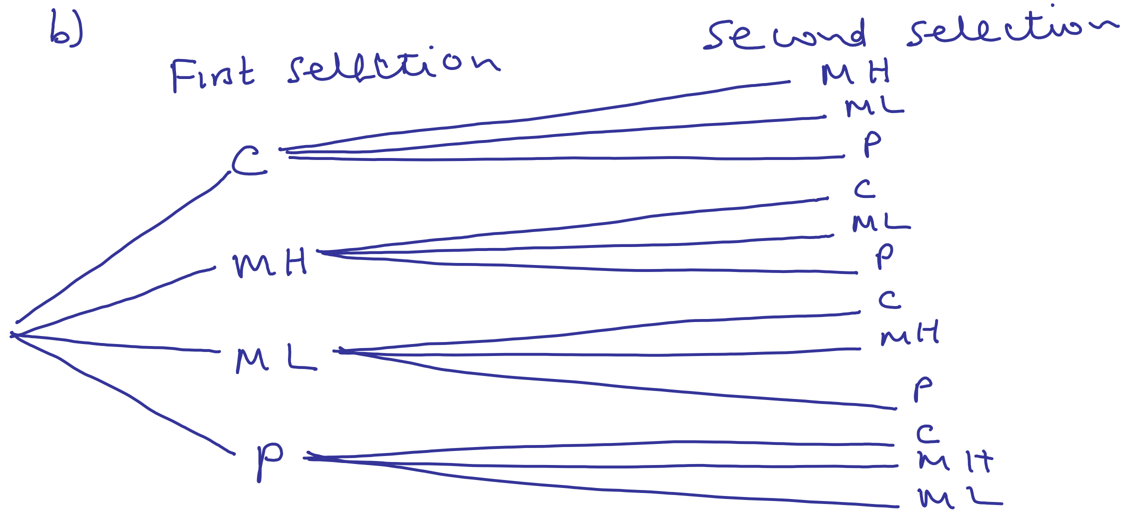
# of points in sample space $= 4 \cdot 3 = 12$

c) P(Christine is selected) $= \frac{6}{12} = \frac{1}{2}$

d) P(neither Mike selected) $= \frac{2}{12} = \frac{1}{6}$

e) P(at least one Mike is selected) $= \frac{10}{12} = \frac{5}{6}$

b)



SAMPLE SPACE

C	MH
C	ML
C	P
MH	C
MH	ML
MH	P
ML	C
ML	MH
ML	P
P	C
P	MH
P	ML

In any probability problem if E is a specific event, then either E happens at least one time or it does not happen at all.

$$P(\text{event happening at least once}) = 1 - P(\text{event does not happen})$$